An Introduction To Topological Data Analysis

This book is an elementary introduction to geometric topology and its applications to chemistry, molecular biology, and cosmology. It does not assume any mathematical or scientific background, sophistication, or even motivation to study mathematics. It is meant to be fun and engaging while drawing students in to learn about fundamental topological and geometric ideas. Though the book can be read and enjoyed by nonmathematicians, college students, or even eager high school students, it is intended to be used as an undergraduate textbook. The book is divided into three parts corresponding to the three areas referred to in the title. Part 1 develops techniques that enable two- and three-dimensional creatures to visualize possible shapes for their universe and to use topological and geometric properties to distinguish one such space from another. Part 2 is an introduction to knot theory with an emphasis on invariants. Part 3 presents applications of topology and geometry to molecular symmetries, DNA, and proteins. Each chapter ends with exercises that allow for better understanding of the material. The style of the book is informal and lively. Though all of the definitions and theorems are explicitly stated, they are given in an intuitive rather than a rigorous form, with several hundreds of figures illustrating the

exposition. This allows students to develop intuition about topology and geometry without getting bogged down in technical details.
A rigorous introduction to geometric and topological inference, for anyone interested in a geometric approach to data science.
This book provides a self-contained introduction to the topology and geometry of surfaces and three-manifolds. The main goal is to describe Thurston's geometrisation of three-manifolds, proved by Perelman in 2002. The book is divided into three parts: the first is devoted to hyperbolic geometry, the second to surfaces, and the third to three-manifolds. It contains complete proofs of Mostow's rigidity, the thick-thin decomposition, Thurston's classification of the diffeomorphisms of surfaces (via Bonahon's geodesic currents), the prime and JSJ decomposition, the topological and geometric classification of Seifert manifolds, and Thurston's hyperbolic Dehn filling Theorem.
"In this chapter, we introduce some of the very basics that are used throughout the book. First, we give the definition of a topological space and related notions of open and closed sets, covers, subspace topology. To connect topology and geometry, we devote a section on metric spaces. Maps such as homeomorphism and homotopy equivalence that play a significant role to relate topological spaces. Certain categories of topological spaces become important for their wide presence in applications. Manifolds are one such category which we

introduce in this chapter. Functions on them satisfying certain conditions are presented as Morse functions. The critical points of such functions relate to the topology of the manifold they are defined on. We introduce these concepts in the smooth setting in this chapter, and later adapt them for the piecewise linear domains frequently used for finite computations. Finally, a section on Notes points out to the history and relevant literature for the concepts delineated in the chapter. It ends with a series of exercises that may be used for teaching a class on the subject both at graduate and undergraduate level"--

This book is a result of a workshop, the 8th of the successful TopoInVis workshop series, held in 2019 in Nyköping, Sweden. The workshop regularly gathers some of the world's leading experts in this field. Thereby, it provides a forum for discussions on the latest advances in the field with a focus on finding practical solutions to open problems in topological data analysis for visualization. The contributions provide introductory and novel research articles including new concepts for the analysis of multivariate and time-dependent data, robust computational approaches for the extraction and approximations of topological structures with theoretical guarantees, and applications of topological scalar and vector field analysis for visualization. The applications span a wide range of scientific areas comprising climate science, material sciences, fluid dynamics, and astronomy. In addition, community efforts with respect to joint software development are reported and discussed.

Computational Topology for Data Analysis

An Introduction with Application to Topological Groups

A Short Course on Topological Insulators

Topological Methods in Data Analysis and Visualization VI

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Manifolds, the higher-dimensional analogs of smooth curves and surfaces, are fundamental objects in modern mathematics. Combining aspects of algebra, topology, and analysis, manifolds have also been applied to classical mechanics, general relativity, and quantum field theory. In this streamlined introduction to the subject, the theory of manifolds is presented with the aim of helping the reader achieve a rapid mastery of the essential topics. By the end of the book the reader should be able to compute, at least for simple spaces, one of the most basic topological invariants of a manifold, its de Rham cohomology. Along the way, the reader acquires the knowledge and skills necessary for further study of geometry and topology. The requisite point-set topology is included in an appendix of twenty pages; other appendices review facts from real analysis and linear algebra. Hints and solutions are provided to many of the exercises and problems. This work may be used as the text for a one-semester graduate or advanced undergraduate course, as well as by students engaged in self-

study. Requiring only minimal undergraduate prerequisites, 'Introduction to Manifolds' is also an excellent foundation for Springer's GTM 82, 'Differential Forms in Algebraic Topology'.

The Handbook of Discrete and Computational Geometry is intended as a reference book fully accessible to nonspecialists as well as specialists, covering all major aspects of both fields. The book offers the most important results and methods in discrete and computational geometry to those who use them in their work, both in the academic world—as researchers in mathematics and computer science—and in the professional world—as practitioners in ?elds as diverse as operations research, molecular biology, and robotics. Discrete geometry has contributed signi?cantly to the growth of discrete mathematics in recent years. This has been fueled partly by the advent of powerful computers and by the recent explosion of activity in the relatively young ?eld of computational geometry. This synthesis between discrete and computational geometry lies at the heart of this Handbook. A growing list of application fields includes combinatorial optimization, computer-aided design, computer graphics, crystallography, data analysis, error-correcting codes, geographic information systems, motion planning, operations research, pattern recognition, robotics, solid modeling, and tomography.

Topology is a large subject with several branches, broadly categorized as algebraic topology, point-set topology, and geometric topology. Point-set topology is the main language for a broad range of mathematical disciplines, while algebraic topology offers as a powerful tool for studying problems in geometry and numerous other areas of mathematics. This book presents the basic concepts of topology, including virtually all of the traditional topics in point-set topology, as well as elementary topics in algebraic topology such as fundamental groups and covering spaces. It also discusses topological groups and transformation groups. When combined with a working knowledge of analysis and algebra, this book offers a valuable resource for advanced undergraduate and beginning graduate students of mathematics specializing in algebraic topology and harmonic analysis.

"Admirably meets the topology requirements for the pregraduate training of research mathematicians." — American Mathematical Monthly Topology, sometimes described as "rubber-sheet geometry," is crucial to modern mathematics and to many other disciplines — from quantum mechanics to sociology. This stimulating introduction to the field will give the student a familiarity with elementary point set topology, including an easy acquaintance with the line and the plane, knowledge often useful in graduate mathematics programs. The book is not a collection of topics, rather it early employs the language of point set topology to define and discuss topological groups. These geometric objects in turn motivate a further discussion of set-theoretic topology and of its applications in function spaces. An introduction to homotopy and the fundamental group then brings the student's new theoretical knowledge to bear on very concrete problems: the calculation of the fundamental group of the circle and a proof of the fundamental theorem of algebra. Finally, the abstract development is brought to satisfying fruition with the classification of topological groups by equivalence under local isomorphism. Throughout the book there is a sustained geometric development — a single thread of reasoning which unifies the topological course. One of the special features of this work is its well-chosen exercises, along with a selection of problems in each chapter that contain interesting applications and further theory. Careful study of the text and diligent performance of the exercises will enable the student to achieve an excellent working knowledge of topology and a useful understanding of its applications. Moreover, the author's unique teaching approach lends an extra dimension of effectiveness to the books: "Of particular interest is the remarkable pedagogy evident in this work. The author converses with the reader on a personal basis. He speaks with him, questions him, challenges him, and — best of all — occasionally leaves him to his own devices." — American Scientist

Computational Algebraic Geometry

Knots, Molecules, and the Universe

Basic Topology

IWCTA 2018, Kochi, India, December 9–11

Geometric and Topological Inference

This solution manual accompanies the first part of the book An Illustrated

Introduction toTopology and Homotopy by the same author. Except for a small number of exercises inthe first few sections, we provide solutions of the (228) odd-numbered problemsappearing in first part of the book (Topology). The primary targets of this manual are thestudents of topology. This set is not disjoint from the set of instructors of topologycourses, who may also find this manual useful as a source of examples, exam problems,etc.

This book provides a concise introduction to topology and is necessary for courses in differential geometry, functional analysis, algebraic topology, etc. Topology is a fundamental tool in most branches of pure mathematics and is also omnipresent in more applied parts of mathematics. Therefore students will need fundamental topological notions already at an early stage in their bachelor programs. While there are already many excellent monographs on general topology, most of them are too large for a first bachelor course. Topology fills this gap and can be either used for self-study or as the basis of a topology course.

Excellent text covers vector fields, plane homology and the Jordan Curve Theorem, surfaces, homology of complexes, more. Problems and exercises. Some knowledge of differential equations and multivariate calculus required.Bibliography. 1979 edition.

This book provides an accessible yet rigorous introduction to topology and

homology focused on the simplicial space. It presents a compact pipeline from the foundations of topology to biomedical applications. It will be of interest to medical physicists, computer scientists, and engineers, as well as undergraduate and graduate students interested in this topic. Features: Presents a practical guide to algebraic topology as well as persistence homology Contains application examples in the field of biomedicine, including the analysis of histological images and point cloud data

Homology is a powerful tool used by mathematicians to study the properties of spaces and maps that are insensitive to small perturbations. This book uses a computer to develop a combinatorial computational approach to the subject. The core of the book deals with homology theory and its computation. Following this is a section containing extensions to further developments in algebraic topology, applications to computational dynamics, and applications to image processing. Included are exercises and software that can be used to compute homology groups and maps. The book will appeal to researchers and graduate students in mathematics, computer science, engineering, and nonlinear dynamics.

Handbook of Discrete and Computational Geometry, Third Edition

Topology for Computing

An Introduction to Algebraic Topology

Theory and Applications
Topological Data Analysis for Genomics and Evolution

This course-based primer provides newcomers to the field with a concise introduction to some of the core topics in the emerging field of topological insulators. The aim is to provide a basic understanding of edge states, bulk topological invariants, and of the bulk--boundary correspondence with as simple mathematical tools as possible. The present approach uses noninteracting lattice models of topological insulators, building gradually on these to arrive from the simplest one-dimensional case (the Su-Schrieffer-Heeger model for polyacetylene) to two-dimensional time-reversal invariant topological insulators (the Bernevig-Hughes-Zhang model for HgTe). In each case the discussion of simple toy models is followed by the formulation of the general arguments regarding topological insulators. The only prerequisite for the reader is a working knowledge in quantum mechanics, the relevant solid state physics background is provided as part of this self-contained text, which is complemented by end-of-chapter problems.

An introduction to geometric and topological methods to analyze large scale biological data; includes statistics and genomic applications.

This book provides an introduction to topology, differential topology, and differential geometry. It is based on manuscripts refined through use in a variety of lecture courses. The first chapter covers elementary results and concepts from point-set topology. An exception is the Jordan Curve Theorem, which is proved for polygonal paths and is intended to give students a first glimpse into the nature of deeper topological problems. The second chapter of the book introduces manifolds and Lie groups, and examines a wide assortment of examples. Further discussion explores tangent bundles, vector bundles, differentials, vector fields, and Lie brackets of vector fields. This discussion is deepened and expanded in the third chapter, which introduces the de Rham cohomology and the oriented integral and gives proofs of the Brouwer Fixed-Point Theorem, the Jordan-Brouwer Separation Theorem, and Stokes's integral formula. The fourth and final chapter is devoted to the fundamentals of differential geometry and traces the development of ideas from curves to submanifolds of Euclidean spaces. Along the way, the book discusses connections and curvature--the central concepts of differential geometry. The discussion culminates with the Gauß equations and the version of Gauß's theorema egregium for submanifolds of arbitrary dimension and codimension. This book is primarily aimed at advanced undergraduates in mathematics and physics and is intended as the template for a one- or two-semester bachelor's course.

Concise treatment covers semitopological groups, locally compact groups, Harr measure, and duality theory and some of its applications. The volume concludes with a chapter that introduces Banach algebras. 1966 edition.

This timely text introduces topological data analysis from scratch, with detailed case studies.

An Introduction to Geographical Information Science

Topology

Topological Data Structures for Surfaces

Elementary Applied Topology

Introduction to Topology

Persistence theory emerged in the early 2000s as a new theory in the area of applied and computational topology. This book provides a broad and modern view of the subject, including its algebraic, topological, and algorithmic aspects. It also elaborates on applications in data analysis. The level of detail of the exposition has been set so as to keep a survey style, while providing sufficient insights into *the proofs so the reader can understand the mechanisms at work. The book is organized into three parts. The first part is dedicated to the foundations of persistence and emphasizes its connection to quiver representation theory. The second part focuses on its connection to applications through a few selected topics. The third part provides perspectives for both the theory and its applications. The book can be used as a text for a course on applied topology or data analysis.*

Combining theoretical and practical aspects of topology, this book provides a comprehensive and self-contained introduction to topological methods for the analysis and visualization of scientific data. Theoretical concepts are presented in a painstaking but intuitive manner, with numerous high-quality color illustrations. Key algorithms for the computation and simplification of topological data representations are described in detail, and their application is carefully demonstrated in a chapter dedicated to concrete use cases. With its fine balance between theory and practice, "Topological Data Analysis for Scientific Visualization" constitutes an appealing introduction to the increasingly important topic of topological data analysis for lecturers, students and researchers.

Combining physics, mathematics and computer science, topological quantum computation is a rapidly expanding research area focused on the exploration of quantum evolutions that are immune to errors. In this book, the author presents a variety of different topics developed together for the first time, forming an excellent introduction to topological quantum computation. The makings of

anyonic systems, their properties and their computational power are presented in a pedagogical way. Relevant calculations are fully explained, and numerous worked examples and exercises support and aid understanding. Special emphasis is given to the motivation and physical intuition behind every mathematical concept. Demystifying difficult topics by using accessible language, this book has broad appeal and is ideal for graduate students and researchers from various disciplines who want to get into this new and exciting research field.
Topology is a relatively young and very important branch of mathematics, which studies the properties of objects that are preserved through deformations, twistings, and stretchings. This book deals with the topology of curves and surfaces as well as with the fundamental concepts of homotopy and homology, and does this in a lively and well-motivated way. This book is well suited for readers who are interested in finding out what topology is all about.
In this broad introduction to topology, the author searches for topological invariants of spaces, together with techniques for their calculating. Students with knowledge of real analysis, elementary group theory, and linear algebra will quickly become familiar with a wide variety of techniques and applications involving point-set, geometric, and algebraic topology. Over 139 illustrations and more than 350 problems of various difficulties help students gain a thorough understanding of the subject.
Topological Data Analysis for Scientific Visualization
Second Edition

Topological Dynamics and Topological Data Analysis
An Illustrated Introduction to Topology and Homotopy Solutions Manual for Part 1
Topology
A Combinatorial Introduction to Topology

In Geography and GIS, surfaces can be analysed and visualised through various data structures, and topological data structures describe surfaces in the form of a relationship between certain surface-specific features. Drawn from many disciplines with a strong applied aspect, this is a research-led, interdisciplinary approach to the creation, analysis and visualisation of surfaces, focussing on topological data structures. Topological Data Structures for Surfaces: an introduction for Geographical Information Science describes the concepts and applications of these data structures. The book focuses on how these data structures can be used to analyse and visualise surface datasets from a range of disciplines such as human geography, computer graphics, metrology, and physical geography. Divided into two Parts, Part I defines the topological surface data structures and explains the various automated methods used for their generation. Part II demonstrates a number of applications of surface networks in diverse fields, ranging from sub-atomic particle collision visualisation to the study of population density patterns. To ensure that the material is accessible, each Part is prefaced by an overview of the techniques and application. Provides GI scientists and geographers with an *accessible overview of current surface topology research. Algorithms are presented and explained with practical examples of their usage. Features an accompanying website developed by the Editor - http://geog.le.ac.uk/sanjayrana/surface-networks/ This book is invaluable for researchers and postgraduate students working in departments of GI Science, Geography and Computer Science. It also constitutes key reference material for Masters students working on surface analysis projects as part of a GI Science or Computer Science programme.*

First course in algebraic topology for advanced undergraduates. Homotopy theory, the duality theorem, relation of topological ideas to other branches of pure mathematics. Exercises and problems. 1972 edition.

The first five chapters of this book form an introductory course in piece wise-linear topology in which no assumptions are made other than basic topological notions. This course would be suitable as a second course in topology with a geometric flavour, to follow a first course in point-set topology, andi)erhaps to be given as a final year undergraduate course. The whole book gives an account of handle theory in a piecewise linear setting and could be the basis of a first year postgraduate lecture or reading course. Some results from algebraic topology are needed for handle theory and these are collected in an appendix. In a second appen dix are listed the properties of Whitehead torsion which are used in the s-cobordism theorem. These appendices should enable a *reader with only basic knowledge to complete the book. The book is also intended to form an introduction to modern geo metric topology as a research subject, a bibliography of research papers being included. We have omitted acknowledgements and references from the main text and have collected these in a set of "historical notes" to be found after the appendices.*

Topological Data Analysis for Scientific VisualizationSpringer

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Computational Topology

An Introduction to Topology and Homotopy

Band Structure and Edge States in One and Two Dimensions

An Introduction to Geometric Topology

Introduction to Geometry and Topology

Combining concepts from topology and algorithms, this book delivers what its title promises: an introduction to the field of computational topology. Starting with motivating problems in both mathematics and computer science and building up from classic topics in geometric and algebraic topology, the third part of the text advances to persistent homology. This point of view is critically important in turning a mostly theoretical field of mathematics into one that is relevant to a multitude of disciplines in the sciences and engineering. The main approach is the discovery of topology through algorithms. The book is ideal for teaching a graduate or advanced undergraduate course in computational topology, as it develops all the background of both the mathematical and algorithmic aspects of the subject from first principles. Thus the text could serve equally well in a course taught in a mathematics department or computer science department.

Algebraandtopology,thetwofundamentaldomainsofmathematics,playcomplem-tary roles. Topology studies continuity and convergence and provides a general framework to study the concept of a limit. Much of topology is devoted to handling in?nite sets and in?nity itself; the methods

developed are qualitative and, in a certain sense, irrational. - gebra studies all kinds of operations and provides a basis for algorithms and calculations. Very often, the methods here are ?nitistic in nature. Because of this difference in nature, algebra and topology have a strong tendency to develop independently, not in direct contact with each other. However, in applications, in higher level domains of mathematics, such as functional analysis, dynamical systems, representation theory, and others, topology and algebra come in contact most naturally. Many of the most important objects of mathematics represent a blend of algebraic and of topologicalstructures. Topologicalfunctionspacesandlineartopologicalspacesingeneral, topological groups and topological ?elds, transformation groups, topological lattices are objects of this kind. Very often an algebraic structure and a topology come naturally together; this is the case when they are both determined by the nature of the elements of the set considered (a group of transformations is a typical example). The rules that describe the relationship between a topology and an algebraic operation are almost always transparentandnatural—th eoperationhastobecontinuous,jointlyorseparately.

Concise work presents topological concepts in clear, elementary fashion, from basics of set-theoretic topology, through topological

theorems and questions based on concept of the algebraic complex, to the concept of Betti groups. Includes 25 figures.
The theory of persistence modules originated in topological data analysis and became an active area of research in algebraic topology. This book provides a concise and self-contained introduction to persistence modules and focuses on their interactions with pure mathematics, bringing the reader to the cutting edge of current research. In particular, the authors present applications of persistence to symplectic topology, including the geometry of symplectomorphism groups and embedding problems. Furthermore, they discuss topological function theory, which provides new insight into oscillation of functions. The book is accessible to readers with a basic background in algebraic and differential topology.
This text explains nontrivial applications of metric space topology to analysis. Covers metric space, point-set topology, and algebraic topology. Includes exercises, selected answers, and 51 illustrations. 1983 edition.
Introduction to Topological Quantum Computation
Computational Homology
An Introduction to Manifolds
Elementary Concepts of Topology
Topological Persistence in Geometry and Analysis

The emerging field of computational topology utilizes theory from topology and the power of computing to solve problems in diverse fields. Recent applications include computer graphics, computer-aided design (CAD), and structural biology, all of which involve understanding the intrinsic shape of some real or abstract space. A primary goal of this book is to present basic concepts from topology and Morse theory to enable a non-specialist to grasp and participate in current research in computational topology. The author gives a self-contained presentation of the mathematical concepts from a computer scientist's point of view, combining point set topology, algebraic topology, group theory, differential manifolds, and Morse theory. He also presents some recent advances in the area, including topological persistence and hierarchical Morse complexes. Throughout, the focus is on computational challenges and on presenting algorithms and data structures when appropriate.

Manifolds play an important role in topology, geometry, complex analysis, algebra, and classical mechanics. Learning manifolds differs from most other introductory mathematics in that the subject matter is often completely unfamiliar. This introduction guides readers by explaining the roles manifolds play in diverse branches of mathematics and physics. The book begins with the basics of general topology and gently moves to manifolds, the fundamental group, and covering spaces.

This book gives an introduction to the mathematics and applications comprising the new field of applied topology. The elements of this subject are surveyed in the context of applications drawn from the biological, economic, engineering, physical, and statistical sciences.

This book collects select papers presented at the International Workshop and Conference on Topology & Applications, held in Kochi, India, from 9–11 December 2018. The book discusses topics on topological dynamical systems and topological data analysis. Topics are ranging from general topology, algebraic topology, differential topology, fuzzy topology, topological dynamical systems, topological groups, linear dynamics, dynamics of operator network topology, iterated function systems and applications of topology. All contributing authors are eminent academicians, scientists, researchers and scholars in their respective fields, hailing from around the world. The book is a valuable resource for researchers, scientists and engineers from both academia and industry.

" . . . that famous pedagogical method whereby one begins with the general and proceeds to the particular only after the student is too confused to understand even that anymore. " Michael Spivak This text was written as an antidote to topology courses such as Spivak It is meant to provide the student with an experience in geomet describes. ric topology. Traditionally, the only topology an undergraduate might see is point-set topology at a fairly abstract level. The next course the average stu dent would take would be a graduate course in algebraic topology, and such courses are commonly very homological in nature, providing quick access to current research, but not developing any intuition or geometric sense. I have tried in this text to provide the undergraduate with a pragmatic introduction to the field, including a sampling from point-set, geometric, and algebraic topology, and trying not to include anything that the student cannot immediately experience. The exercises are to be considered as an in tegral part of the text and, ideally, should be addressed when they are met, rather than at the end of a block of material. Many of them are quite easy and are intended to give the student practice working with the definitions and digesting the current topic before proceeding. The appendix provides a brief survey of the group theory needed.

Introduction to Piecewise-Linear Topology

Topology in Biology

A Geometric Introduction to Topology

Introduction to Topological Manifolds

An Introduction

This text is an introduction to topology and homotopy. Topics are integrated into a coherent whole and developed slowly so students will not be overwhelmed.

Topology of Surfaces

Persistence Theory: From Quiver Representations to Data Analysis

Topological Data Analysis with Applications

Computational Topology for Biomedical Image and Data Analysis

Introduction to Topological Groups