Calculus Of Variations

This comprehensive text provides all information necessary for an introductory course on the calculus of variations and optimal control theory. Following a thorough discussion of the basic problem, including sufficient conditions for optimality, the theory and techniques are extended to problems with a free end point, a free boundary, auxiliary and inequality constraints, leading to a study of optimal control theory.

This textbook provides a comprehensive introduction to the classical and modern calculus of variations, serving as a useful reference to advanced undergraduate and graduate students as well as researchers in the field. Starting from ten motivational examples, the book begins with the most important aspects of the classical theory, including the Direct Method, the Euler-Lagrange equation, Lagrange multipliers, Noether's Theorem and some regularity theory. Based on the efficient Young measure approach, the author then discusses the vectorial theory of integral functionals, including quasiconvexity, polyconvexity, and relaxation. In the second part, more recent material such as rigidity in differential inclusions, microstructure, convex integration, singularities in measures, functionals defined on functions of bounded variation (BV), and Γ-convergence for phase transitions and homogenization are explored. While predominantly designed as a textbook for lecture courses on the calculus of variations, this book can also serve as the basis for a reading seminar or as a companion for self-study. The reader is assumed to be familiar with basic vector analysis, functional analysis, Sobolev spaces, and measure theory, though most of the preliminaries are also recalled in the appendix.

This text is basically divided into two parts. Chapters 1–4 include background material, basic theorems and isoperimetric problems. Chapters 5–12 are devoted to applications, geometrical optics, particle dynamics, the theory of elasticity, electrostatics, quantum mechanics, and other topics. Exercises in each chapter. 1952 edition.

Introduction to the Calculus of Variations and Control with Modern Applications provides the fundamental background required to develop rigorous necessary conditions that are the starting points for theoretical and numerical approaches to modern variational calculus and control problems. The book also presents some classical sufficient conditions and discusses the importance of distinguishing between the necessary and sufficient conditions. In the first part of the text, the author develops the calculus of variations and provides complete proofs of the main results. He explains how the ideas behind the proofs are essential to the development of modern optimization and control theory. Focusing on optimal control problems, the second part shows how optimal control is a natural extension of the classical calculus of variations to more complex problems. By emphasizing the basic ideas and their mathematical development, this book gives you the foundation to use these mathematical tools to then tackle new problems. The text moves from simple to more complex problems, allowing you to see how the fundamental theory can be modified to address more difficult and advanced challenges. This approach helps you understand how to deal with future problems and applications in a realistic work environment.

Introduction to the Calculus of Variations

The Calculus of Variations

Representation, Relaxation, and Homogenization

An Intuitive Introduction

This two-volume treatise is a standard reference in the field. It pays special attention to the historical aspects and the origins partly in applied problems—such as those of geometric optics—of parts of the theory. It contains an introduction to each chapter, section, and subsection and an overview of the relevant literature in the footnotes and bibliography. It also includes an index of the examples used throughout the book.

- Serves as an excellent introduction to the calculus of variations - Useful to researchers in different fields of mathematics who want to get a concise but broad introduction to the subject - Includes more than 70 exercises with solutions

The purpose of the calculus of variations is to find optimal solutions to engineering problems whose optimum may be a certain quantity, shape, or function. Applied Calculus of Variations for Engineers addresses this important mathematical area applicable to many engineering disciplines. Its unique, application-oriented approach sets it apart from the theoretical treatises of most texts, as it is aimed at enhancing the engineer's understanding of the topic. This Second Edition text: Contains new chapters discussing analytic solutions of variational problems and Lagrange-Hamilton equations of motion in depth Provides new

sections detailing the boundary integral and finite element methods and their calculation techniques Includes enlightening new examples, such as the compression of a beam, the optimal cross section of beam under bending force, the solution of Laplace’s equation, and Poisson’s equation with various methods Applied Calculus of Variations for Engineers, Second Edition extends the collection of techniques aiding the engineer in the application of the concepts of the calculus of variations.

This is the first of two books on methods and techniques in the calculus of variations. Contemporary arguments are used throughout the text to streamline and present in a unified way classical results, and to provide novel contributions at the forefront of the theory. This book addresses fundamental questions related to lower semicontinuity and relaxation of functionals within the unconstrained setting, mainly in L^p spaces. It prepares the ground for the second volume where the variational treatment of functionals involving fields and their derivatives will be undertaken within the framework of Sobolev spaces. This book is self-contained. All the statements are fully justified and proved, with the exception of basic results in measure theory, which may be found in any good textbook on the subject. It also contains several exercises. Therefore,it may be used both as a graduate textbook as well as a reference text for researchers in the field. Irene Fonseca is the Mellon College of Science Professor of Mathematics and is currently the Director of the Center for Nonlinear Analysis in the Department of Mathematical Sciences at Carnegie Mellon University. Her research interests lie in the areas of continuum mechanics, calculus of variations, geometric measure theory and partial differential equations. Giovanni Leoni is also a professor in the Department of Mathematical Sciences at Carnegie Mellon University. He focuses his research on calculus of variations, partial differential equations and geometric measure theory with special emphasis on applications to problems in continuum mechanics and in materials science.

Calculus of Variations and Optimal Control Theory

An Introduction to the Calculus of Variations

An Introduction to the One-Dimensional Theory with Examples and Exercises

Calculus of Variations

Publisher Description

Suitable for advanced undergraduate and graduate students of mathematics, physics, or engineering, this introduction to the calculus of variations focuses on variational problems involving one independent variable. It also discusses more advanced topics such as the inverse problem, eigenvalue problems, and Noether’s theorem. The text includes numerous examples along with problems to help students consolidate the material.

The calculus of variations is one of the oldest subjects in mathematics, and it is very much alive and still evolving. Besides its mathematical importance and its links to other branches of mathematics, such as geometry or differential equations, it is widely used in physics, engineering, economics and biology. This book serves both as a guide to the expansive existing literature and as an aid to the non-specialist — mathematicians, physicists, engineers, students or researchers — in discovering the subject's most important problems, results and techniques. Despite the aim of addressing non-specialists, mathematical rigor has not been sacrificed; most of the theorems are either fully proved or proved under more stringent conditions. In this new edition, several new exercises have been added. The book, containing a total of 119 exercises with detailed solutions, is well designed for a course at both undergraduate and graduate levels.

The calculus of variations is a subject whose beginning can be precisely dated. It might be said to begin at the moment that Euler coined the name calculus of variations but this is, of course, not the true moment of inception of the subject. It would not have been unreasonable if I had gone back to the set of isoperimetric problems considered by Greek mathemati cians such as Zenodorus (c. 200 B. C.) and preserved by Pappus (c. 300 A. D.). I have not done this since these problems were solved by geometric means. Instead I have arbitrarily chosen to begin with Fermat's elegant principle of least time. He used this principle in 1662 to show how a light ray was refracted at the interface between two optical media of different densities. This analysis of Fermat seems to me especially appropriate as a starting point: He used the methods of the calculus to minimize the time of passage cif a light ray through the two media, and his method was adapted by John Bernoulli to solve the brachystochrone problem. There have been several other histories of the subject, but they are now hopelessly archaic. One by Robert Woodhouse appeared in 1810 and another by Isaac Todhunter in 1861.

Functional Analysis, Calculus of Variations and Optimal Control

Classical Mechanics with Calculus of Variations and Optimal Control

Calculus of Variations with Applications

Topics on Geometrical Evolution Problems and Degree Theory

This book is intended for a first course in the calculus of variations, at the senior or beginning graduate level. The reader will learn methods for finding functions that maximize or minimize integrals. The text lays out important necessary and sufficient conditions for extrema in historical order, and it illustrates these conditions with numerous worked-out examples from mechanics, optics, geometry, and other fields. The exposition starts with simple integrals containing a single independent variable, a single dependent variable, and a single derivative, subject to weak variations, but steadily moves on to more advanced topics, including multivariate problems, constrained extrema, homogeneous problems, problems with variable endpoints, broken extremals, strong variations, and sufficiency conditions. Numerous line drawings clarify the mathematics. Each chapter ends with recommended readings that introduce the student to the relevant scientific literature and with exercises that consolidate understanding.

Functional analysis owes much of its early impetus to problems that arise in the calculus of variations. In turn, the methods developed there have been applied to optimal control, an area that also requires new tools, such as nonsmooth analysis. This self-contained textbook gives a complete course on all these topics. It is written by a leading specialist who is also a noted expositor. This book provides a thorough introduction to functional analysis and includes many novel elements as well as the standard topics. A short course on nonsmooth analysis and geometry completes the first half of the book whilst the second half concerns the calculus of variations and optimal control. The author provides a comprehensive course on these subjects, from their inception through to the present. A notable feature is the inclusion of recent, unifying developments on regularity, multiplier rules, and the Pontryagin maximum principle, which appear here for the first time in a textbook. Other major themes include existence and Hamilton-Jacobi methods. The

many substantial examples, and the more than three hundred exercises, treat such topics as viscosity solutions, nonsmooth Lagrangians, the logarithmic Sobolev inequality, periodic trajectories, and systems theory. They also touch lightly upon several fields of application: mechanics, economics, resources, finance, control engineering. Functional Analysis, Calculus of Variations and Optimal Control is intended to support several different courses at the first-year or second-year graduate level, on functional analysis, on the calculus of variations and optimal control, or on some combination. For this reason, it has been organized with customization in mind. The text also has considerable value as a reference. Besides its advanced results in the calculus of variations and optimal control, its polished presentation of certain other topics (for example convex analysis, measurable selections, metric regularity, and nonsmooth analysis) will be appreciated by researchers in these and related fields.

Calculus of variations is one of the most important mathematical tools of great scientific significance used by scientistis and engineers. Unfortunately, a few books that are available are written at a level which is not easily comprehensible for postgraduate students.This book, written by a highly respected academic, presents the materials in a lucid manner so as to be within the easy grasp of the students with some background in calculus, differential equations and functional analysis. The aim is to give a thorough and systematic analysis of various aspects of calculus of variations.

First truly up-to-date treatment offers a simple introduction to optimal control, linear-quadratic control design, and more. Broad perspective features numerous exercises, hints, outlines, and appendixes, including a practical discussion of MATLAB. 2005 edition.

Lectures on the Calculus of Variations

Introduction to the Calculus of Variations and Control with Modern Applications

A Concise Introduction

Introduction To The Calculus of Variations And Its Applications

Clear, rigorous introductory treatment covers applications to geometry, dynamics, and physics. It focuses upon problems with one independent variable, connecting abstract theory with its use in concrete problems. 1962 edition.

This book is developed for the study of vectorial problems in the calculus of variations. The subject is a very active one and almost half of the book consists of new material. This is a new edition of the earlier book published in 1989 and it is suitable for graduate students. The book has been updated with some new material and examples added. Applications are included.

In recent years there has been a considerable renewal of interest in the clas sical problems of the calculus of variations, both from the point of view of mathematics and of applications. Some of the most powerful tools for proving existence of minima for such problems are known as direct methods. They are often the only available ones, particularly for vectorial problems. It is the aim of this book to present them. These methods were introduced by Tonelli, following earlier work of Hilbert and Lebesgue. Although there are excellent books on calculus of variations and on direct methods, there are recent important developments which cannot be found in these books; in particular, those dealing with vector valued functions and relaxation of non convex problems. These two last ones are important in appli cations to nonlinear elasticity, optimal design . . . . In these fields the variational methods are particularly effective. Part of the mathematical developments and of the renewal of interest in these methods finds its motivations in nonlinear elasticity. Moreover, one of the recent important contributions to nonlinear analysis has been the study of the behaviour of nonlinear functionals un der various types of convergence, particularly the weak convergence. Two well studied theories have now been developed, namely f-convergence and compen sated compactness. They both include as a particular case the direct methods of the calculus of variations, but they are also, both, inspired and have as main examples these direct methods.

In this highly regarded text for advanced undergraduate and graduate students, the author develops the calculus of variations both for its intrinsic interest and for its powerful applications to modern mathematical physics. Topics include first and second variations of an integral, generalizations, isoperimetrical problems, least action, special relativity, elasticity, more. 1963 edition.

A History of the Calculus of Variations from the 17th through the 19th Century

Modern Methods in the Calculus of Variations

Local and Global Theory

Pioneering modern treatise studies the development of the subject from Euler to Hilbert, addressing basic problems with sufficient generality and rigor to provide a sound introduction for serious study. 1904 edition.

The aim of the present book is to give a systematic treatment of the inverse problem of the calculus of variations, i.e. how to recognize whether a system of differential equations can be treated as a system for extremals of a variational functional (the Euler-Lagrange equations), using contemporary geometric methods. Selected applications in geometry, physics, optimal control, and general relativity are also considered. The book includes the following chapters: - Helmholtz conditions and the method of controlled Lagrangians (Bloch, Krupka, Zenkov) - The Sonin-Douglas's problem (Krupka) - Inverse variational problem and symmetry in action: The Ostrogradskyj relativistic third order dynamics (Matsyuk.) - Source forms and their variational completion (Voicu) - First-order variational sequences and the inverse problem of the calculus of variations (Urban, Volna) - The inverse problem of the calculus of variations on Grassmann fibrations (Urban).

Introduction to the Calculus of Variations and Control with Modern Applications provides the fundamental background required to develop rigorous necessary conditions that are the starting points for theoretical and numerical approaches to modern variational calculus and control problems. The book also presents some classical sufficient conditions a

This concise text offers both professionals and students an introduction to the fundamentals and standard methods of the calculus of variations. In addition to surveys of problems with fixed and movable boundaries, it explores highly practical direct methods for the solution of variational problems. Topics include the method of variation in problems with fixed boundaries; variational problems with movable boundaries and other problems; sufficiency conditions for an extremum; variational problems of constrained extrema; and direct methods of solving variational problems. Each chapter features numerous illustrative problems, and solutions appear at the end.

Unbounded Functionals in the Calculus of Variations

With Applications to Physics and Engineering

Mechanics, Control and Other Applications
Calculus of Variations and Partial Differential Equations

Fresh, lively text serves as a modern introduction to the subject, with applications to the mechanics of systems with a finite number of degrees of freedom. Ideal for math and physics students.

Applications-oriented introduction to variational theory develops insight and promotes understanding of specialized books and research papers. Suitable for advanced undergraduate and graduate students as a primary or supplementary text. 1969 edition.

Provides a thorough understanding of calculus of variations and prepares readers for the study of modern optimal control theory. Selected variational problems and over 400 exercises. Bibliography. 1969 edition.

At the summer school in Pisa in September 1996, Luigi Ambrosio and Norman Dancer each gave a course on the geometric problem of evolution of a surface by mean curvature, and degree theory with applications to PDEs respectively. This self-contained presentation accessible to PhD students bridged the gap between standard courses and advanced research on these topics. The resulting book is divided accordingly into 2 parts, and neatly illustrates the 2-way interaction of problems and methods. Each of the courses is augmented and complemented by additional short chapters by other authors describing current research problems and results.

L^p Spaces

Third Edition

Calculus of Variations - With Applications to Physics and Engineering

Direct Methods in the Calculus of Variations

From the reviews: "...the book contains a wealth of material essential to the researcher concerned with multiple integral variational problems and with elliptic partial differential equations. The book not only reports the researches of the author but also the contributions of his contemporaries in the same and related fields. The book undoubtedly will become a standard reference for researchers in these areas. ...The book is addressed mainly to mature mathematical analysts. However, any student of analysis will be greatly rewarded by a careful study of this book." M. R. Hestenes in Journal of Optimization Theory and Applications "The work intertwines in masterly fashion results of classical analysis, topology, and the theory of manifolds and thus presents a comprehensive treatise of the theory of multiple integral variational problems." L. Schmetterer in Monatshefte für Mathematik "The book is very clearly exposed and contains the last modern theory in this domain. A comprehensive bibliography ends the book." M. Coroi-Nedeleu in Revue Roumaine de Mathématiques Pures et Appliquées

This textbook offers a concise yet rigorous introduction to calculus of variations and optimal control theory, and is a self-contained resource for graduate students in engineering, applied mathematics, and related subjects. Designed specifically for a one-semester course, the book begins with calculus of variations, preparing the ground for optimal control. It then gives a complete proof of the maximum principle and covers key topics such as the Hamilton-Jacobi-Bellman theory of dynamic programming and linear-quadratic optimal control. Calculus of Variations and Optimal Control Theory also traces the historical development of the subject and features numerous exercises, notes and references at the end of each chapter, and suggestions for further study. Offers a concise yet rigorous introduction Requires limited background in control theory or advanced mathematics Provides a complete proof of the maximum principle Uses consistent notation in the exposition of classical and modern topics Traces the historical development of the subject Solutions manual (available only to teachers) Leading universities that have adopted this book include: University of Illinois at Urbana-Champaign ECE 553: Optimum Control Systems Georgia Institute of Technology ECE 6553: Optimal Control and Optimization University of Pennsylvania ESE 680: Optimal Control Theory University of Notre Dame EE 60565: Optimal Control

This is an intuitively motivated presentation of many topics in classical mechanics and related areas of control theory and calculus of variations. All topics throughout the book are treated with zero tolerance for unrevealing definitions and for proofs which leave the reader in the dark. Some areas of particular interest are: an extremely short derivation of the ellipticity of planetary orbits; a statement and an explanation of the "tennis racket paradox"; a heuristic explanation (and a rigorous treatment) of the gyroscopic effect; a revealing equivalence between the dynamics of a particle and statics of a spring; a short geometrical explanation of Pontryagin's Maximum Principle, and more. In the last chapter, aimed at more advanced readers, the Hamiltonian and the momentum are compared to forces in a certain static problem. This gives a palpable physical meaning to some seemingly abstract concepts and theorems. With minimal prerequisites consisting of basic calculus and basic undergraduate physics, this book is suitable for courses from an undergraduate to a beginning graduate level, and for a mixed audience of mathematics, physics and engineering students. Much of the enjoyment of the subject lies in solving almost 200 problems in this book.

Calculus of VariationsWith Applications to Physics and EngineeringCourier Corporation

CALCULUS OF VARIATIONS WITH APPLICATIONS
Applied Calculus of Variations for Engineers
Calculus of Variations I
The Inverse Problem of the Calculus of Variations

This clear and concise textbook provides a rigorous introduction to the calculus of variations, depending on functions of one variable and their first derivatives. It is based on a translation of a German edition of the book Variationsrechnung (Vieweg+Teubner Verlag, 2010), translated and updated by the author himself. Topics include: the Euler-Lagrange equation for one-dimensional variational problems, with and without constraints, as well as an introduction to the direct methods. The book targets students who have a solid background in calculus and linear algebra, not necessarily in functional analysis. Some advanced mathematical tools, possibly not familiar to the reader, are given along with proofs in the appendix. Numerous figures, advanced problems and proofs, examples, and exercises with solutions accompany the book, making it suitable for self-study. The book will be particularly useful for beginning graduate students from the physical, engineering, and mathematical sciences with a rigorous theoretical background.

This text is in two sections. the first part dealing with, background material, basic theorems and isoperimetric problems. The second part devoted to applications, geometrical optics, particle dynamics, he theory of elasticity, electrostatics, quantum mechanics and much more. Many of the earliest books, particularly those dating back to the 1900s and before, are now extremely scarce and increasingly expensive. We are republishing these classic works in affordable, high quality, modern editions, using the original text and artwork.

This 1927 book constitutes Scottish mathematician Andrew Russell Forsyth's attempt at a systematic exposition of the calculus of variations.

This book provides a comprehensive discussion on the existence and regularity of minima of regular integrals in the calculus of variations and of solutions to elliptic partial differential equations and systems of the second order. While direct methods for the existence of solutions are well known and have been widely used in the last century, the regularity of the minima was always obtained by means of the Euler equation as a part of the general theory of partial differential equations. In this book, using the notion of the quasi-minimum introduced by Giaquinta and the author, the direct methods are extended to the regularity of the minima of functionals in the calculus of variations, and of solutions to partial differential equations. This unified treatment offers a substantial economy in the assumptions, and permits a deeper understanding of the nature of the regularity and singularities of the solutions. The book is essentially self-contained, and requires only a general knowledge of the elements of Lebesgue integration theory.

Multiple Integrals in the Calculus of Variations

A First Course in the Calculus of Variations

Over the last few decades, research in elastic-plastic torsion theory, electrostatic screening, and rubber-like nonlinear elastomers has pointed the way to some interesting new classes of minimum problems for energy functionals of the calculus of variations. This advanced-level monograph addresses these issues by developing the framework of a gener