

*Commutative Algebra With A View Toward Algebraic Geometry Corrected 3rd Printing*

**This textbook offers a thorough, modern introduction into commutative algebra. It is intended mainly to serve as a guide for a course of one or two semesters, or for self-study. The carefully selected subject matter concentrates on the concepts and results at the center of the field. The book maintains a constant view on the natural geometric context, enabling the reader to gain a deeper understanding of the material. Although it emphasizes theory, three chapters are devoted to computational aspects. Many illustrative examples and exercises enrich the text.**

**An introduction to abstract algebraic geometry, with the only prerequisites being results from commutative algebra, which are stated as needed, and some elementary topology. More than 400 exercises distributed throughout the book offer specific examples as well as more specialised topics not treated in the main text, while three appendices present brief accounts of some areas of current research. This book can thus be used as textbook for an introductory course in algebraic geometry following a basic graduate course in algebra. Robin Hartshorne studied algebraic geometry with Oscar Zariski and David Mumford at Harvard, and with J.-P. Serre and A. Grothendieck in Paris. He is the author of "Residues and Duality", "Foundations of Projective Geometry", "Ample Subvarieties of Algebraic Varieties", and numerous research titles.**

**Algebraic geometry is a fascinating branch of mathematics that combines methods from both, algebra and geometry. It transcends the limited scope of pure algebra by means of geometric construction principles. Moreover, Grothendieck's schemes invented in the late 1950s allowed the application of algebraic-geometric methods in fields that formerly seemed to be far away from geometry, like algebraic number theory. The new techniques paved the way to spectacular progress such as the proof of Fermat's Last Theorem by Wiles and Taylor. The scheme-theoretic approach to algebraic geometry is explained for non-experts. More advanced readers can use the book to broaden their view on the subject. A separate part deals with the necessary prerequisites from commutative algebra. On a whole, the book provides a very accessible and self-contained introduction to algebraic geometry, up to a quite advanced level. Every chapter of the book is preceded by a motivating introduction with an informal discussion of the contents. Typical examples and an abundance of exercises illustrate each section. This way the book is an excellent solution for learning by yourself or for complementing knowledge that is already present. It can equally be used as a convenient source for courses and seminars or as supplemental literature.**

**Introducing the representation theory of groups and finite dimensional algebras, first studying basic non-commutative ring theory, this book covers the necessary background on elementary homological algebra and representations of groups up to block theory. It further discusses vertices, defect groups, Green and Brauer correspondences and Clifford theory. Whenever possible the statements are presented in a general setting for more general algebras, such as symmetric finite dimensional algebras over a field. Then, abelian and derived categories are introduced in detail and are used to explain stable module categories, as well as derived categories and their main invariants and links between them. Group theoretical applications of these theories are given - such as the structure of blocks of cyclic defect groups - whenever appropriate. Overall, many methods from the representation theory of algebras are introduced. Representation Theory assumes only the most basic knowledge of linear algebra, groups, rings and fields and guides the reader in the use of categorical equivalences in the representation theory of groups and algebras. As the book is based on lectures, it will be accessible to any graduate student in algebra and can be used for self-study as well as for classroom use.**

**Exercises in Modules and Rings**

**Homological and Computational Methods in Commutative Algebra**

**Introduction To Commutative Algebra**

**Proceedings of a Workshop held in Salvador, Brazil, Aug. 8-17, 1988**

**Undergraduate Commutative Algebra**

*In this chapter we are largely influenced in our choice of material by the demands of the rest of the book. However, we take the view that this is an opportunity for the student to grasp basic categorical notions which permeate so much of mathematics today, including, of course, algebraic topology, so that we do not allow ourselves to be rigidly restricted by our immediate objectives. A reader totally unfamiliar with category theory may find it easiest to restrict his first reading of Chapter II to Sections 1 to 6; large parts of the book are understandable with the material presented in these sections. Another reader, who had already met many examples of categorical formulations and concepts might, in fact, prefer to look at Chapter II before reading Chapter I. Of course the reader thoroughly familiar with category theory could, in principal, omit Chapter II, except perhaps to familiarize himself with the notations employed. In Chapter III we begin the proper study of homological algebra by looking in particular at the group  $\text{Ext}_A(A, B)$ , where  $A$  and  $B$  are  $A$ -modules. It is shown how this group*

can be calculated by means of a projective presentation of  $A$ , or an injective presentation of  $B$ ; and how it may also be identified with the group of equivalence classes of extensions of the quotient module  $A$  by the submodule  $B$ .

Translated from the popular French edition, this book offers a detailed introduction to various basic concepts, methods, principles, and results of commutative algebra. It takes a constructive viewpoint in commutative algebra and studies algorithmic approaches alongside several abstract classical theories. Indeed, it revisits these traditional topics with a new and simplifying manner, making the subject both accessible and innovative. The algorithmic aspects of such naturally abstract topics as Galois theory, Dedekind rings, Prüfer rings, finitely generated projective modules, dimension theory of commutative rings, and others in the current treatise, are all analysed in the spirit of the great developers of constructive algebra in the nineteenth century. This updated and revised edition contains over 350 well-arranged exercises, together with their helpful hints for solution. A basic knowledge of linear algebra, group theory, elementary number theory as well as the fundamentals of ring and module theory is required. *Commutative Algebra: Constructive Methods* will be useful for graduate students, and also researchers, instructors and theoretical computer scientists.

This book can be understood as a model for teaching commutative algebra, and takes into account modern developments such as algorithmic and computational aspects. As soon as a new concept is introduced, the authors show how the concept can be worked on using a computer. The computations are exemplified with the computer algebra system Singular, developed by the authors. Singular is a special system for polynomial computation with many features for global as well as for local commutative algebra and algebraic geometry. The book includes a CD containing Singular as well as the examples and procedures explained in the book.

The Abel Symposium 2009 "Combinatorial aspects of Commutative Algebra and Algebraic Geometry", held at Voss, Norway, featured talks by leading researchers in the field. This is the proceedings of the Symposium, presenting contributions on syzygies, tropical geometry, Boij-Söderberg theory, Schubert calculus, and quiver varieties. The volume also includes an introductory survey on binomial ideals with applications to hypergeometric series, combinatorial games and chemical reactions. The contributions pose interesting problems, and offer up-to-date research on some of the most active fields of commutative algebra and algebraic geometry with a combinatorial flavour.

*Fuzzy Commutative Algebra*

*Commutative Ring Theory*

*A Singular Introduction to Commutative Algebra*

*Ideals, Varieties, and Algorithms*

*Combinatorial Aspects of Commutative Algebra and Algebraic Geometry*

The central theme of this volume is commutative algebra, with emphasis on special graded algebras, which are increasingly of interest in problems of algebraic geometry, combinatorics, and computer algebra. Most of the papers have partly survey character, but are research-oriented, aiming at classification and structural results.

There is no shortage of books on Commutative Algebra, but the present book is different. Most books are monographs, with extensive coverage. There is one notable exception: Atiyah and Macdonald's 1969 classic. It is a clear, concise, and efficient textbook, aimed at beginners, with a good selection of topics. So it has remained popular. However, its age and flaws are obvious. So there is need for an updated and improved version, which the present book aims to be.

Commutative algebra is a rapidly growing subject that is developing in many different directions. This volume presents several of the most recent results from various areas related to commutative algebra, Noetherian and non-Noetherian commutative algebra. This volume contains a collection of invited survey articles by some of the leading experts in the field. The authors of these chapters have been carefully selected for their important contributions to an area of commutative-algebraic research. Some topics presented in the volume include: generalizations of cyclic zero divisor graphs, class semigroups, forcing algebras, syzygy bundles, tight closure, Gorenstein dimensions, tensor products of algebras over fields, as well as many others. This book is intended for researchers and graduate students interested in studying the many topics related to commutative algebra.

Now in paperback, this advanced text on Cohen-Macaulay rings has been updated and expanded.

*Graduate Algebra*

*The Use of Ultraproducts in Commutative Algebra*

*Computational Commutative Algebra 1*

*Algebraic Geometry and Commutative Algebra*

*Commutative Algebra: Constructive Methods*

This book provides an introduction to the basics and recent developments of commutative algebra. A glance at the contents of the first five chapters shows that the topics covered are ones that usually are included in any commutative algebra text. However, the contents of this book differ significantly from most commutative algebra texts: namely, its treatment of the Dedekind-Mertens formula, the (small) finitistic dimension of a ring, Gorenstein rings, valuation overrings and the valuative dimension, and Nagata rings. Going further, Chapter 6 presents  $w$ -modules over commutative rings as they can be most commonly used by torsion theory and multiplicative ideal theory. Chapter 7 deals with

multiplicative ideal theory over integral domains. Chapter 8 collects various results of the pullbacks, especially Milnor squares and D+M constructions, which are probably the most important example-generating machines. In Chapter 9, coherent rings with finite weak global dimensions are probed, and the local ring of weak global dimension two is elaborated on by combining homological tricks and methods of star operation theory. Chapter 10 is devoted to the Grothendieck group of a commutative ring. In particular, the Bass-Quillen problem is discussed. Finally, Chapter 11 aims to introduce relative homological algebra, especially where the related concepts of integral domains which appear in classical ideal theory are defined and investigated by using the class of Gorenstein projective modules. Each section of the book is followed by a selection of exercises of varying degrees of difficulty. This book will appeal to a wide readership from graduate students to academic researchers who are interested in studying commutative algebra.

This ACM volume deals with tackling problems that can be represented by data structures which are essentially matrices with polynomial entries, mediated by the disciplines of commutative algebra and algebraic geometry. The discoveries stem from an interdisciplinary branch of research which has been growing steadily over the past decade. The author covers a wide range, from showing how to obtain deep heuristics in a computation of a ring, a module or a morphism, to developing means of solving nonlinear systems of equations - highlighting the use of advanced techniques to bring down the cost of computation. Although intended for advanced students and researchers with interests both in algebra and computation, many parts may be read by anyone with a basic abstract algebra course.

From the Preface: "We have preferred to write a self-contained book which could be used in a basic graduate course of modern algebra. It is also with an eye to the student that we have tried to give full and detailed explanations in the proofs... We have also tried, this time with an eye to both the student and the mature mathematician, to give a many-sided treatment of our topics, not hesitating to offer several proofs of one and the same result when we thought that something might be learned, as to methods, from each of the proofs." Grothendieck's beautiful theory of schemes permeates modern algebraic geometry and underlies its applications to number theory, physics, and applied mathematics. This simple account of that theory emphasizes and explains the universal geometric concepts behind the definitions. In the book, concepts are illustrated with fundamental examples, and explicit calculations show how the constructions of scheme theory are carried out in practice.

Computational Methods in Commutative Algebra and Algebraic Geometry

Commutative View

Representation Theory

A Homological Algebra Point of View

In Honor of Masayoshi Nagata

This book presents algorithmic tools for algebraic geometry, with experimental applications. It also introduces Macaulay 2, a computer algebra system supporting research in algebraic geometry, commutative algebra, and their applications. The algorithmic tools presented here are designed to serve readers wishing to bring such tools to bear on their own problems. The first part of the book covers Macaulay 2 using concrete applications; the second emphasizes details of the mathematics.

Exploring ultraproducts of Noetherian local rings from an algebraic perspective, this volume illustrates the many ways they can be used in commutative algebra. The text includes an introduction to tight closure in characteristic zero, a survey of flatness criteria, and more.

Introductory account of commutative algebra, aimed at students with a background in basic algebra.

This is a comprehensive review of commutative algebra, from localization and primary decomposition through dimension theory, homological methods, free resolutions and duality, emphasizing the origins of the ideas and their connections with other parts of mathematics. The book gives a concise treatment of Grobner basis theory and the constructive methods in commutative algebra and algebraic geometry that flow from it. Many exercises included.

Constructive Commutative Algebra

Algebraic Geometry

The Geometry of Schemes

A Course in Homological Algebra

Cohen-Macaulay Rings

Originally published in 1985, this classic textbook is an English translation of Einführung in die kommutative Algebra und algebraische Geometrie. As part of the Modern Birkhäuser Classics series, the publisher is proud to make Introduction to Commutative Algebra and Algebraic Geometry available to a wider audience. Aimed at students who have taken a basic course in algebra, the goal of the text is to present important results concerning the representation of algebraic varieties as intersections of the least possible number of hypersurfaces and—a closely related problem—with the most economical generation of ideals in Noetherian rings. Along the way, one encounters many basic concepts of commutative algebra and algebraic geometry and proves many facts which can then serve as a basic stock for a deeper study of these subjects.

For those looking for an introduction to the area of commutative algebra, this book opens all the right doors and provides a clarity of understanding that all

will welcome.

Recent developments are covered Contains over 100 figures and 250 exercises Includes complete proofs

The main goal of this book is to find the constructive content hidden in abstract proofs of concrete theorems in Commutative Algebra, especially in well-known theorems concerning projective modules over polynomial rings (mainly the Quillen-Suslin theorem) and syzygies of multivariate polynomials with coefficients in a valuation ring. Simple and constructive proofs of some results in the theory of projective modules over polynomial rings are also given, and light is cast upon recent progress on the Hermite ring and Gröbner ring conjectures. New conjectures on unimodular completion arising from our constructive approach to the unimodular completion problem are presented. Constructive algebra can be understood as a first preprocessing step for computer algebra that leads to the discovery of general algorithms, even if they are sometimes not efficient. From a logical point of view, the dynamical evaluation gives a constructive substitute for two highly nonconstructive tools of abstract algebra: the Law of Excluded Middle and Zorn's Lemma. For instance, these tools are required in order to construct the complete prime factorization of an ideal in a Dedekind ring, whereas the dynamical method reveals the computational content of this construction. These lecture notes follow this dynamical philosophy.

(Mostly) Commutative Algebra

Dedicated to Winfried Bruns on the Occasion of his 70th Birthday

Local Algebra

Noncommutative View

Steps in Commutative Algebra

***This volume collects contributions by leading experts in the area of commutative algebra related to the INdAM meeting "Homological and Computational Methods in Commutative Algebra" held in Cortona (Italy) from May 30 to June 3, 2016 . The conference and this volume are dedicated to Winfried Bruns on the occasion of his 70th birthday. In particular, the topics of this book strongly reflect the variety of Winfried Bruns' research interests and his great impact on commutative algebra as well as its applications to related fields. The authors discuss recent and relevant developments in algebraic geometry, commutative algebra, computational algebra, discrete geometry and homological algebra. The book offers a unique resource, both for young and more experienced researchers seeking comprehensive overviews and extensive bibliographic references.***

***Interest in commutative algebra has surged over the past decades. In order to survey and highlight recent developments in this rapidly expanding field, the Centre de Recerca Matemàtica in Bellaterra organized a ten-days Summer School on Commutative Algebra in 1996. Lectures were presented by six high-level specialists, L. Avramov (Purdue), M.K. Green (UCLA), C. Huneke (Purdue), P. Schenzel (Halle), G. Valla (Genova) and W.V. Vasconcelos (Rutgers), providing a fresh and extensive account of the results, techniques and problems of some of the most active areas of research. The present volume is a synthesis of the lectures given by these authors. Research workers as well as graduate students in commutative algebra and nearby areas will find a useful overview of the field and recent developments in it. Reviews "All six articles are at a very high level; they provide a thorough survey of results and methods in their subject areas, illustrated with algebraic or geometric examples." - Acta Scientiarum Mathematicarum Avramov lecture: "... it contains all the major results [on infinite free resolutions], it explains carefully all the different techniques that apply, it provides complete proofs (...). This will be extremely helpful for the novice as well as the experienced." - Mathematical reviews Huneke lecture: "The topic is tight closure, a theory developed by M. Hochster and the author which has in a short time proved to be a useful and powerful tool. (...) The paper is extremely well organized, written, and motivated." - Zentralblatt MATH Schenzel lecture: "... this paper is an excellent introduction to applications of local cohomology." - Zentralblatt MATH Valla lecture: "... since he is an acknowledged expert on Hilbert functions and since his interest has been so broad, he has done a superb job in giving the readers a lively picture of the theory." - Mathematical reviews Vasconcelos lecture: "This is a very useful survey on invariants of modules over noetherian rings, relations between them, and how to compute them." - Zentralblatt MATH***

***This is an English translation of the now classic "Algbre Locale - Multiplicités" originally published by Springer as LNM 11. It gives a short account of the main theorems of commutative algebra, with emphasis on modules, homological methods and intersection multiplicities. Many modifications to the original French text have been made for this English edition, making the text easier to read, without changing its intended informal character.***

***This book is an expanded text for a graduate course in commutative algebra, focusing on the algebraic underpinnings of algebraic geometry and of number theory. Accordingly, the theory of affine algebras is featured, treated both directly and via the theory of Noetherian and Artinian modules, and the theory of graded algebras is included to provide the foundation for projective varieties. Major topics include the theory of modules over a principal ideal domain, and its application to matrix theory (including the Jordan decomposition), the Galois theory of field extensions, transcendence degree, the prime spectrum of an algebra, localization, and the classical theory of Noetherian and Artinian rings. Later chapters include some algebraic theory of elliptic curves (featuring the Mordell-Weil theorem) and valuation theory, including local fields. One feature of the book is an extension of the text through a series of appendices. This permits the inclusion of more advanced material, such as transcendental field extensions, the discriminant and resultant, the theory of Dedekind domains, and basic theorems of rings of algebraic integers. An***

**extended appendix on derivations includes the Jacobian conjecture and Makar-Limanov's theory of locally nilpotent derivations. Grobnerbases can be found in another appendix. Exercises provide a further extension of the text. The book can be used both as a textbook and as a reference source.**

**A Course in Commutative Algebra**

**Computations in Algebraic Geometry with Macaulay 2**

**Commutative Algebra**

**Foundations of Commutative Rings and Their Modules**

Written at a level appropriate to undergraduates, this book covers such topics as the Hilbert Basis Theorem, the Nullstellensatz, invariant theory, projective geometry, and dimension theory. Contains a new section on Axiom and an update about MAPLE, Mathematica and REDUCE.

This introduction to polynomial rings, Gröbner bases and applications bridges the gap in the literature between theory and actual computation. It details numerous applications, covering fields as disparate as algebraic geometry and financial markets. To aid in a full understanding of these applications, more than 40 tutorials illustrate how the theory can be used. The book also includes many exercises, both theoretical and practical.

This book is the first to be devoted entirely to fuzzy abstract algebra. It presents an up-to-date version of fuzzy commutative algebra, and focuses on the connection between L-subgroups of a group, and L-subfields of a field. In particular, an up-to-date treatment of nonlinear systems of fuzzy intersection equations is given. Contents:L-Subsets and L-SubgroupsL-Subgroups of Abelian GroupsL-Subrings and L-IdealsL-SubmodulesL-SubfieldsStructure of L-Subrings and L-IdealsAlgebraic L-Varieties and Intersection EquationsL-SubspacesGalois Theory and Group L-Subalgebras Readership: Pure mathematicians. Keywords:Fuzzy Subsets;Fuzzy Subgroups;Fuzzy Subrings;Fuzzy Ideals;Fuzzy Submodules ;Fuzzy Subfields;Fuzzy Varieties;Fuzzy Subspaces;Fuzzy Galois Theory;Fuzzy Group SubalgebrasReviews: "The book is self-contained ... This will serve as a nice reference book for researchers in the field. It may also be used as a good text for an advanced graduate course. There are a good number of exercises at the end of each chapter of the book." Mathematical Reviews

Algebraic Geometry and Commutative Algebra in Honor of Masayoshi Nagata presents a collection of papers on algebraic geometry and commutative algebra in honor of Masayoshi Nagata for his significant contributions to commutative algebra. Topics covered range from power series rings and rings of invariants of finite linear groups to the convolution algebra of distributions on totally disconnected locally compact groups. The discussion begins with a description of several formulas for enumerating certain types of objects, which may be tabular arrangements of integers called Young tableaux or some types of monomials. The next chapter explains how to establish these enumerative formulas, with emphasis on the role played by transformations of determinantal polynomials and recurrence relations satisfied by them. The book then turns to several applications of the enumerative formulas and universal identity, including including enumerative proofs of the straightening law of Doubilet-Rota-Stein and computations of Hilbert functions of polynomial ideals of certain determinantal loci. Invariant differentials and quaternion extensions are also examined, along with the moduli of Todorov surfaces and the classification problem of embedded lines in characteristic  $p$ . This monograph will be a useful resource for practitioners and researchers in algebra and geometry.

with a View Toward Algebraic Geometry

With a View Toward Algebraic Geometry

Combinatorial Commutative Algebra

Basic Commutative Algebra

The Abel Symposium 2009

This textbook, set for a one or two semester course in commutative algebra, provides an introduction to commutative algebra at the postgraduate and research levels. The main prerequisites are familiarity with groups, rings and fields. Proofs are self-contained. The book will be useful to beginners and experienced researchers alike. The material is so arranged that the beginner can learn through self-study or by attending a course. For the experienced researcher, the book may serve to present new perspectives on some well-known results, or as a reference.

This book is a companion volume to Graduate Algebra: Commutative View (published as volume 73 in this series). The main and most important feature of the book is that it presents a unified approach to many important topics, such as group theory, ring theory, Lie algebras, and gives conceptual proofs of many basic results of noncommutative algebra. There are also a number of major results in noncommutative algebra that are usually found only in technical works, such as Zelmanov's proof of the restricted Burnside problem in group theory, word problems in groups, Tits's alternative in algebraic groups, PI algebras, and many of the roles that Coxeter diagrams play in algebra. The first half of the book can serve as a one-semester course on noncommutative algebra, whereas the remaining part of the book describes some of the major

directions of research in the past 100 years. The main text is extended through several appendices, which permits the inclusion of more advanced material, and numerous exercises. The only prerequisite for using the book is an undergraduate course in algebra; whenever necessary, results are quoted from Graduate Algebra: Commutative View.

This book stems from lectures on commutative algebra for 4th-year university students at two French universities (Paris and Rennes). At that level, students have already followed a basic course in linear algebra and are essentially fluent with the language of vector spaces over fields. The topics introduced include arithmetic of rings, modules, especially principal ideal rings and the classification of modules over such rings, Galois theory, as well as an introduction to more advanced topics such as homological algebra, tensor products, and algebraic concepts involved in algebraic geometry. More than 300 exercises will allow the reader to deepen his understanding of the subject. The book also includes 11 historical vignettes about mathematicians who contributed to commutative algebra.

This book explores commutative ring theory, an important a foundation for algebraic geometry and complex analytical geometry.

Projective Modules Over Polynomial Rings and Dynamical Gröbner Bases

Noetherian and Non-Noetherian Perspectives

Introduction to Commutative Algebra and Algebraic Geometry

A Term of Commutative Algebra

Commutative Algebra I

First Published in 2018. Routledge is an imprint of Taylor & Francis, an Informa company.

Commutative Algebra with a View Toward Algebraic Geometry Springer Science & Business Media

This volume offers a compendium of exercises of varying degree of difficulty in the theory of modules and rings. It is the companion volume to GTM 189. All exercises are solved in full detail. Each section begins with an introduction giving the general background and the theoretical basis for the problems that follow.

An Introduction to Computational Algebraic Geometry and Commutative Algebra

Six Lectures on Commutative Algebra

Finite Projective Modules