Hyperbolic Geometry Springer

This text on geometry is devoted to various central geometrical topics including: graphs of functions, transformations, (non-)Euclidean geometries, curves and surfaces as well as their applications in a variety of disciplines. This book presents elementary methods for analytical modeling and demonstrates the potential for symbolic computational tools to support the development of analytical solutions. The author systematically examines several powerful tools of MATLAB® including 2D and 3D animation of geometric images with shadows and colors and transformations using matrices. With over 150 stimulating exercises and problems, this text integrates traditional differential and non-Euclidean geometries with more current computer systems in a practical and user-friendly format. This text is an excellent classroom resource or self-study reference for undergraduate students in a variety of disciplines.

Based on a course taught for years at Oxford, this book offers a concise exposition of the central ideas of general relativity. The focus is on the chain of reasoning that leads to the relativistic theory from the analysis of distance and time measurements in the presence of gravity, rather than on the underlying mathematical structure. Includes links to recent developments, including theoretical work and observational evidence, to encourage further study.

This textbook offers a geometric perspective on special relativity, bridging Euclidean space, hyperbolic space, and Einstein's spacetime in one accessible, self-contained volume. Using tools tailored to undergraduates, the author explores Euclidean and non-Euclidean geometries, gradually building from intuitive to abstract spaces. By the end, readers will have encountered a range of topics, from isometries to the Lorentz–Minkowski plane, building an understanding of how geometry can be used to model special relativity. Beginning with intuitive spaces, such as the Euclidean plane and the sphere, a structure theorem for isometries is introduced that serves as a foundation for increasingly sophisticated topics, such as the hyperbolic plane and the Lorentz–Minkowski plane. By gradually introducing tools throughout, the author offers readers an accessible pathway to visualizing increasingly abstract geometric concepts. Numerous exercises are also included with selected solutions provided. Geometry: from Isometries to Special Relativity offers a unique approach to non-Euclidean geometries, culminating in a mathematical model for special relativity. The focus on isometries offers undergraduates an accessible progression from the intuitive to abstract; instructors will appreciate the complete instructor solutions manual available online. A background in elementary calculus is assumed.

In recent years, geometry has played a lesser role in undergraduate courses than it has ever done. Nevertheless, it still plays a leading role in mathematics at a higher level. Its central role in the history of mathematics has never been disputed. It is important, therefore, to introduce some geometry into university syllabuses. There are several ways of doing this, it can be incorporated into existing courses that are primarily devoted to other topics, it can be taught at a first year level or it can be taught in higher level courses devoted to differential geometry or to more classical topics. These notes are intended to fill a rather obvious gap in the literature. It treats the classical topics of Euclidean, projective and hyperbolic geometry but uses the material commonly taught to undergraduates: linear algebra, group theory, metric spaces and complex analysis. The notes are based on a course whose aim was two fold, firstly, to introduce the students to some geometry and secondly to deepen their understanding of topics that they have already met. What is required from the earlier material is a familiarity with the main ideas, specific topics that are used are usually redone.

Modeling of Curves and Surfaces with MATLAB®

Geometry and Spectra of Compact Riemann Surfaces

Growth, Amenability, and Random Walks

An Elementary Account of Galilean Geometry and the Galilean Principle of Relativity

Foundations of Hyperbolic Manifolds

This text is intended to serve as an introduction to the geometry of the action of discrete groups of Mobius transformations. The subject matter has now been studied with changing points of emphasis for over a hundred years, the most recent developments being connected with the theory of 3-manifolds: see, for example, the papers of Poincare [77] and Thurston [101]. About 1940, the now well-known (but virtually unobtainable) Fenchel-Nielsen manuscript appeared. Sadly, the manuscript never appeared in print, and this more modest text attempts to display at least some of the beautiful geo metrical ideas to be found in that manuscript, as well as some more recent material. The text has been written with the conviction that geometrical explana tions are essential for a full understanding of the material and that however simple a matrix proof might seem, a geometric proof is almost certainly more profitable. Further, wherever possible, results should be stated in a form that is invariant under conjugation, thus making the intrinsic nature of the result more apparent. Despite the fact that the subject matter is concerned with groups of isometries of hyperbolic geometry, many publications rely on Euclidean estimates and geometry. However, the recent developments have again emphasized the need for hyperbolic geometry, and I have included a comprehensive chapter on analytical (not axiomatic) hyperbolic geometry. It is hoped that this chapter will serve as a "dictionary" offormulae in plane hyperbolic geometry and as such will be of interest and use in its own right.

Presented as an engaging discourse, this textbook invites readers to delve into the historical origins and uses of geometry. The narrative traces the influence of Euclid's system of geometry, as developed in his classic text The Elements, through the Arabic period, the modern era in the West, and up to twentieth century mathematics. Axioms and proof methods used by mathematicians from those periods are explored alongside the problems in Euclidean geometry that lead to their work. Students cultivate skills applicable to much of modern mathematics through sections that integrate concepts like projective and hyperbolic geometry with representative proof-based exercises. For its sophisticated account of ancient to modern geometries, this text assumes only a year of college mathematics as it builds towards its conclusion with algebraic curves and quaternions. Euclid's work has affected geometry for thousands of years, so this text has something to offer to anyone who wants to broaden their *appreciation for the field.*

This book is the result of many years of research in Non-Euclidean Geometries and Geometry of Lie groups, as well as teaching at Moscow State University (1947- 1949), Azerbaijan State University (Baku) (1950-1955), Kolomna Pedagogical Col lege (1955-1970), Moscow Pedagogical University (1971-1990), and Pennsylvania State University (1990-1995). My first books on Non-Euclidean Geometries and Geometry of Lie groups were written in Russian and published in Moscow: Non-Euclidean Geometries (1955) [Ro1] , Multidimensional Spaces (1966) [Ro2] , and Non-Euclidean Spaces (1969) [Ro3]. In [Ro1] I considered non-Euclidean geometries in the broad sense, as geometry of simple Lie groups, since classical non-Euclidean geometries, hyperbolic and elliptic, are geometries of simple Lie groups of classes Bn and D , and geometries of complex n and quaternionic Hermitian elliptic and hyperbolic spaces are geometries of simple Lie groups of classes An and en. [Ro1] contains an exposition of the geometry of classical real non-Euclidean spaces and their interpretations as hyperspheres with identified antipodal points in Euclidean or pseudo-Euclidean spaces, and in projective and conformal spaces. Numerous interpretations of various spaces different from our usual space allow us, like stereoscopic vision, to see many traits of these spaces absent in the usual space.

This book is an exposition of the theoretical foundations of hyperbolic manifolds. It is intended to be used both as a textbook and as a reference. Particular emphasis has been placed on readability and completeness of ar gument. The treatment of the material is for the most part elementary and self-contained. The reader is assumed to have a basic knowledge of algebra and topology at the first-year graduate level of an American university. The book is divided into three parts. The first part, consisting of Chap ters 1-7, is concerned with hyperbolic geometry and basic properties of discrete groups of isometries of hyperbolic space. The main results are the existence theorem for discrete reflection groups, the Bieberbach theorems, and Selberg's lemma. The second part, consisting of Chapters 8-12, is de voted to the theory of hyperbolic manifolds. The main results are Mostow's rigidity theorem and the determination of the structure of geometrically finite hyperbolic manifolds. The third part, consisting of Chapter 13, in tegrates the first two parts in a development of the theory of hyperbolic orbifolds. The main results are the construction of the universal orbifold covering space and Poincare's fundamental polyhedron theorem.

The Geometry of Discrete Groups

Geometry of Surfaces

Geometry

Perspectives on Projective Geometry

Modern Geometry with Applications

Pressley assumes the reader knows the main results of multivariate calculus and concentrates on the theory of the study of surfaces. Used for courses on surface geometry, it

includes intersting and in-depth examples and goes into the subject in great detail and vigour. The book will cover three-dimensional Euclidean space only, and takes the whole book to cover the material and treat it as a subject in its own right.
Lie groups and Lie algebras have become essential to many parts of mathematics and theoretical physics, with Lie algebras a central object of interest in their own right. This book provides an elementary introduction to Lie algebras based on a lecture course given to fourth-year undergraduates. The only prerequisite is some linear algebra and an appendix summarizes the main facts that are needed. The treatment is kept as simple as possible with no attempt at full generality. Numerous worked examples and exercises are provided to test understanding, along with more demanding problems, several of which have solutions. Introduction to Lie Algebras covers the core material required for almost all other work in Lie theory and provides a self-study guide suitable for undergraduate students in their final year and graduate students and researchers in mathematics and theoretical physics.
This book is unique in that it looks at geometry from 4 different viewpoints - Euclid-style axioms, linear algebra, projective geometry, and groups and their invariants Approach makes the subject accessible to readers of all mathematical tastes, from the visual to the algebraic Abundantly supplemented with figures and exercises
This book provides an introduction to the main geometric structures that are carried by compact surfaces, with an emphasis on the classical theory of Riemann surfaces. It first covers the prerequisites, including the basics of differential forms, the Poincaré Lemma, the Morse Lemma, the classification of compact connected oriented surfaces, Stokes' Theorem, fixed point theorems and rigidity theorems. There is also a novel presentation of planar hyperbolic geometry. Moving on to more advanced concepts, it covers topics such as Riemannian metrics, the isometric torsion-free connection on vector fields, the Ansatz of Koszul, the Gauss-Bonnet Theorem, and integrability. These concepts are then used for the study of Riemann surfaces. One of the focal points is the Uniformization Theorem for compact surfaces, an elementary proof of which is given via a property of the energy functional. Among numerous other results, there is also a proof of Chow's Theorem on compact holomorphic submanifolds in complex projective spaces. Based on lecture courses given by the author, the book will be accessible to undergraduates and graduates interested in the analytic theory of Riemann surfaces.
The Foundations of Geometry

Geometry: Euclid and Beyond
Euclidean, Hyperbolic, and Projective Geometries
An Introduction
The Four Pillars of Geometry

Focussing on the geometry of hyperbolic manifolds, the aim here is to provide an exposition of some fundamental results, while being as self-contained, complete, detailed and unified as possible. Following some classical material on the hyperbolic space and the Teichmüller space, the book centers on the two fundamental results: Mostow's rigidity theorem (including a complete proof, following Gromov and Thurston) and Margulis' lemma. These then form the basis for studying Chabauty and geometric topology; a unified exposition is given of Wang's theorem and the Jorgensen-Thurston theory; and much space is devoted to the 3D case: a complete and elementary proof of the hyperbolic surgery theorem, based on the representation of three manifolds as glued ideal tetrahedra.

Bridging the gap between novice and expert, the aim of this book is to present in a self-contained way a number of striking examples of current diophantine problems to which Arakelov geometry has been or may be applied. Arakelov geometry can be seen as a link between algebraic geometry and diophantine geometry. Based on lectures from a summer school for graduate students, this volume consists of 12 different chapters, each written by a different author. The first chapters provide some background and introduction to the subject. These are followed by a presentation of different applications to arithmetic geometry. The final part describes the recent application of Arakelov geometry to Shimura varieties and the proof of an averaged version of Colmez's conjecture. This book thus blends initiation to fundamental tools of Arakelov geometry with original material corresponding to current research. This book will be particularly useful for graduate students and researchers interested in the connections between algebraic geometry and number theory. The prerequisites are some knowledge of number theory and algebraic geometry.

This book consists of 16 surveys on Thurston's work and its later development. The authors are mathematicians who were strongly influenced by Thurston's publications and ideas. The subjects discussed include, among others, knot theory, the topology of 3-manifolds, circle packings, complex projective structures, hyperbolic geometry, Kleinian groups, foliations, mapping class groups, Teichmüller theory, anti-de Sitter geometry, and co-Minkowski geometry. The book is addressed to researchers and students who want to learn about Thurston's wide-ranging mathematical ideas and their impact. At the same time, it is a tribute to Thurston, one of the greatest geometers of all time, whose work extended over many fields in mathematics and who had a unique way of perceiving forms and patterns, and of communicating and writing mathematics.

Thoroughly updated, featuring new material on important topics such as hyperbolic geometry in higher dimensions and generalizations of hyperbolicity Includes full solutions for all exercises Successful first edition sold over 800 copies in North America

Elementary Differential Geometry
Lectures on Hyperbolic Geometry
A Metric Approach with Models
Geometry: from Isometries to Special Relativity
General Relativity

This book is concerned with discontinuous groups of motions of the unique connected and simply connected Riemannian 3-manifold of constant curva ture -1, which is traditionally called hyperbolic 3-space. This space is the 3-dimensional instance of an analogous Riemannian manifold which exists uniquely in every dimension n :::: 2. The hyperbolic spaces appeared first in the work of Lobachevski in the first half of the 19th century. Very early in the last century the group of isometries of these spaces was studied by Steiner, when he looked at the group generated by the inversions in spheres. The ge ometries underlying the hyperbolic spaces were of fundamental importance since Lobachevski, Bolyai and Gauß had observed that they do not satisfy the axiom of parallels. Already in the classical works several concrete coordinate models of hy perbolic 3-space have appeared. They make explicit computations possible and also give identifications of the full group of motions or isometries with well-known matrix groups. One such model, due to H. Poincare, is the upper 3 half-space IH in JR . The group of isometries is then identified with an exten sion of index 2 of the group PSL(2,

Recently there has been considerable interest in developing techniques based on number theory to attack problems of 3-manifolds; Contains many examples and lots of problems; Brings together much of the existing literature of Kleinian groups in a clear and concise way; At present no such text exists

This textbook is a comprehensive and yet accessible introduction to non-Euclidean Laguerre geometry, for which there exists no previous systematic presentation in the literature. Moreover, we present new results by demonstrating all essential features of Laguerre geometry on the example of checkerboard incircular nets. Classical (Euclidean) Laguerre geometry studies oriented hyperplanes, oriented hyperspheres, and their oriented contact in Euclidean space. We describe how this can be generalized to arbitrary Cayley-Klein spaces, in particular hyperbolic and elliptic space, and study the corresponding groups of Laguerre transformations. We give an introduction to Lie geometry and describe how these Laguerre geometries can be obtained as subgeometries. As an application of two-dimensional Lie and Laguerre geometry we study the properties of checkerboard incircular nets.

Hyperbolic Manifolds and Discrete Groups is at the crossroads of several branches of mathematics: hyperbolic geometry, discrete groups, 3-dimensional topology, geometric group theory, and complex analysis. The main focus throughout the text is on the "Big Monster," i.e., on Thurston’s hyperbolization theorem, which has not only completely changes the landscape of 3-dimensinal topology and Kleinian group theory but is one of the central results of 3-dimensional topology. The book is fairly self-contained, replete with beautiful illustrations, a rich set of examples of key concepts, numerous exercises, and an extensive bibliography and index. It should serve as an ideal graduate course/seminar text or as a comprehensive reference.

Arakelov Geometry and Diophantine Applications

Introduction to Hyperbolic Geometry

Notes on Geometry

Topological, Differential and Conformal Geometry of Surfaces

Introduction to Lie Algebras

This introduction to modern geometry differs from other books in the field due to its emphasis on applications and its discussion of special relativity as a major example of a non-Euclidean geometry. Additionally, it covers the two important areas of non-Euclidean geometry, spherical geometry and projective geometry, as well as emphasising transformations, and conics and planetary orbits. Much emphasis is placed on applications throughout the book, which motivate the topics, and many additional applications are given in the exercises. It makes an excellent introduction for those who need to know how geometry is used in addition to its formal theory.

Geometry: A Metric Approach with Models, imparts a real feeling for Euclidean and non-Euclidean (in particular, hyperbolic) geometry. Intended as a rigorous first course, the book introduces and develops the various axioms slowly, and then, in a departure from other texts, continually illustrates the major definitions and axioms with two or three models, enabling the reader to picture the idea more clearly. The second edition has been expanded to include a selection of expository exercises. Additionally, the authors have designed software with computational problems to accompany the text. This software may be obtained from George Parker.

There are many technical and popular accounts, both in Russian and in other languages, of the non-Euclidean geometry of Lobachevsky and Bolyai, a few of which are listed in the Bibliography. This geometry, also called hyperbolic geometry, is part of the required subject matter of many mathematics departments in universities and teachers' colleges-a reflec tion of the view that familiarity with the elements of hyperbolic geometry is a useful part of the background of future high school teachers. Much attention is paid to hyperbolic geometry by school mathematics clubs. Some mathematicians and educators concerned with reform of the high school curriculum believe that the required part of the curriculum should include elements of hyperbolic geometry, and that the optional part of the curriculum should include a topic related to hyperbolic geometry. I The broad interest in hyperbolic geometry is not surprising. This interest has little to do with mathematical and scientific applications of hyperbolic geometry, since the applications (for instance, in the theory of automorphic functions) are rather specialized, and are likely to be encountered by very few of the many students who conscientiously study (and then present to examiners) the definition of parallels in hyperbolic geometry and the special features of configurations of lines in the hyperbolic plane. The principal reason for the interest in hyperbolic geometry is the important fact of "non-uniqueness" of geometry; of the existence of many geometric systems.

The text that comprises this volume is a collection of surveys and original works from experts in the fields of algebraic number theory, analytic number theory, harmonic analysis, and hyperbolic geometry. A portion of the collected contributions have been developed from lectures given at the "International Conference on the Occasion of the 60th Birthday of S. J. Patterson", held at the University Göttingen, July 27-29 2009. Many of the included chapters have been contributed by invited participants. This volume presents and investigates the most recent developments in various key topics in analytic number theory and several related areas of mathematics. The volume is intended for graduate students and researchers of number theory as well as applied mathematicians interested in this broad field.

Harmonic Analysis and Number Theory

Geometric Group Theory

A Guided Tour Through Real and Complex Geometry

In the Tradition of Thurston

Festschrift for S. J. Patterson

Projective geometry is one of the most fundamental and at the same time most beautiful branches of geometry. It can be considered the common foundation of many other geometric disciplines like Euclidean geometry, hyperbolic and elliptic geometry or even relativistic space-time geometry. This book offers a comprehensive introduction to this fascinating field and its applications. In particular, it explains how metric concepts may be best understood in projective terms. One of the major themes that appears throughout this book is the beauty of the interplay between geometry, algebra and combinatorics. This book can especially be used as a guide that explains how geometric objects and operations may be most elegantly expressed in algebraic terms, making it a valuable resource for mathematicians, as well as for computer scientists and physicists. The book is based on the author’s experience in implementing geometric software and includes hundreds of high-quality illustrations.

This book is an introduction to hyperbolic and differential geometry that provides material in the early chapters that can serve as a textbook for a standard upper division course on hyperbolic geometry. For that material, the students need to be familiar with calculus and linear algebra and willing to accept one advanced theorem from analysis without proof. The book goes well beyond the standard course in later chapters, and there is enough material for an honors course, or for supplementary reading. Indeed, parts of the book have been used for both kinds of courses. Even some of what is in the early chapters would surely not be nec essary for a standard course. For example, detailed proofs are given of the Jordan Curve Theorem for Polygons and of the decomposability of poly gons into triangles, These proofs are included for the sake of completeness, but the results themselves are so believable that most students should skip the proofs on a first reading. The axioms used are modern in character and more "user friendly" than the traditional ones. The familiar real number system is used as an in gredient rather than appearing as a result of the axioms. However, it should not be thought that the geometric treatment is in terms of models: this is an axiomatic approach that is just more convenient than the traditional ones.

This book offers a unique opportunity to understand the essence of one of the great thinkers of western civilization. A guided reading of Euclid's Elements leads to a critical discussion and rigorous modern treatment of Euclid's geometry and its more recent descendants, with complete proofs. Topics include the introduction of coordinates, the theory of area, history of the parallel postulate, the various non-Euclidean geometries, and the regular and semi-regular polyhedra.

Inspired by classical geometry, geometric group theory has in turn provided a variety of applications to geometry, topology, group theory, number theory and graph theory. This carefully written textbook provides a rigorous introduction to this rapidly evolving field whose methods have proven to be powerful tools in neighbouring fields such as geometric topology. Geometric group theory is the study of finitely generated groups via the geometry of their associated Cayley graphs. It turns out that the essence of the geometry of such groups is captured in the key notion of quasi-isometry, a large-scale version of isometry whose invariants include growth types, curvature conditions, boundary constructions, and amenability. This book covers the foundations of quasi-geometry of groups at an advanced undergraduate level. The subject is illustrated by many elementary examples, outlooks on applications, as well as an extensive collection of exercises.

Topics in Groups and Geometry

Graph Drawing

Geometries and Groups

Finite Blaschke Products and Their Connections

This monograph is a self-contained introduction to the geometry of Riemann Surfaces of constant curvature –1 and their length and eigenvalue spectra. It focuses on two subjects: the geometric theory of compact Riemann surfaces of genus greater than one, and the relationship of the Laplace operator with the geometry of such surfaces. Research workers and graduate students interested in compact Riemann surfaces will find here a number of useful tools and insights to apply to their investigations.

The geometry of surfaces is an ideal starting point for learning geometry, for, among other reasons, the theory of surfaces of constant curvature has maximal connectivity with the rest of mathematics. This text provides the student with the knowledge of a geometry of greater scope than the classical geometry taught today, which is no longer an adequate basis for mathematics or physics, both of which are becoming increasingly geometric. It includes exercises and informal discussions.

An introduction to complex analysis for students with some knowledge of complex numbers from high school. It contains sixteen chapters, the first eleven of which are aimed at an upper division undergraduate audience. The remaining five chapters are designed to complete the coverage of all background necessary for passing PhD qualifying exams in complex analysis. Topics studied include Julia sets and the Mandelbrot set, Dirichlet series and the prime number theorem, and the uniformization theorem for Riemann surfaces, with emphasis placed on the three geometries: spherical, euclidean, and hyperbolic. Throughout, exercises range from the very simple to the challenging. The book is based on lectures given by the author at several universities, including UCLA, Brown University, La Plata, Buenos Aires, and the Universidad Autonomo de Valencia, Spain.

This volume constitutes the refereed proceedings of the 19th International Symposium on Graph Drawing, GD 2010, held in Eindhoven, The Netherlands, during September 2011. The 34 revised full papers presented together with 3 revised short and 6 poster papers were carefully reviewed and selected from 88 submissions. Furthermore, the proceedings contain the abstracts of two invited talks and to commemorate Kozo Sugiyama and his pioneering research in graph drawing, the proceedings include an obituary. A unique and fun part of the symposium is the Graph Drawing Contest, which is part of the Graph Drawing Challenge. This year was the 18th edition. A report on the contest is included at the end of the proceedings.

Fine Structures of Hyperbolic Diffeomorphisms

Geometry Through History

Groups Acting on Hyperbolic Space

Hyperbolic Manifolds and Discrete Groups

The Foundations of Geometry and the Non-Euclidean Plane

This book provides a detailed exposition of a wide range of topics in geometric group theory, inspired by Gromov’s pivotal work in the 1980s. It includes classical theorems on nilpotent groups and solvable groups, a fundamental study of the growth of groups, a detailed look at asymptotic cones, and a discussion of related subjects including filters and ultrafilters, dimension theory, hyperbolic geometry, amenability, the Burnside problem, and random walks on groups. The results are unified under the common theme of Gromov’s theorem, namely that finitely generated groups of polynomial growth are virtually nilpotent. This beautiful result gave birth to a fascinating new area of research which is still active today. The purpose of the book is to collect these naturally related results together in one place, most of which are scattered throughout the literature, some of them appearing here in book form for the first time. In this way, the connections between these topics are revealed, providing a pleasant introduction to geometric group theory based on ideas surrounding Gromov's theorem. The book will be of interest to mature undergraduate and graduate students in mathematics who are familiar with basic group theory and topology, and who wish to learn more about geometric, analytic, and probabilistic aspects of infinite groups.

The study of hyperbolic systems is one of the core themes of modern dynamical systems. This book plays an important role in filling a gap in the present literature on hyperbolic dynamics and is highly recommended for all PhD students interested in this field.

Hyperbolic GeometrySpringer Science & Business Media

Adopts a user-friendly approach, with an emphasis on worked examples and exercises, rather than abstract theory The computer algebra and graphical package MAPLE is used to illustrate many of the ideas and provides an additional aid to teaching and learning Supplementary material, including detailed solutions to exercises and MAPLE worksheets, is available via the web

Non-Euclidean Laguerre Geometry and Incircular Nets

Contributions in Analytic and Algebraic Number Theory

Geometry and Topology
19th International Symposium, GD 2011, Eindhoven, The Netherlands, September 21-23, 2011, Revised Selected Papers
A Simple Non-Euclidean Geometry and Its Physical Basis

This monograph offers an introduction to finite Blaschke products and their connections to complex analysis, linear algebra, operator theory, matrix analysis, and other fields. Old favorites such as the Carathéodory approximation and the Pick interpolation theorems are featured, as are many topics that have never received a modern treatment, such as the Bohr radius and Ritt's theorem on decomposability. Deep connections to hyperbolic geometry are explored, as are the mapping properties, zeros, residues, and critical points of finite Blaschke products. In addition, model spaces, rational functions with real boundary values, spectral mapping properties of the numerical range, and the Darlington synthesis problem from electrical engineering are also covered. Topics are carefully discussed, and numerous examples and illustrations highlight crucial ideas. While thorough explanations allow the reader to appreciate the beauty of the subject, relevant exercises following each chapter improve technical fluency with the material. With much of the material previously scattered throughout mathematical history, this book presents a cohesive, comprehensive and modern exposition accessible to undergraduate students, graduate students, and researchers who have familiarity with complex analysis.

This book is devoted to the theory of geometries which are locally Euclidean, in the sense that in small regions they are identical to the geometry of the Euclidean plane or Euclidean 3-space. Starting from the simplest examples, we proceed to develop a general theory of such geometries, based on their relation with discrete groups of motions of the Euclidean plane or 3-space; we also consider the relation between discrete groups of motions and crystallography. The description of locally Euclidean geometries of one type shows that these geometries are themselves naturally represented as the points of a new geometry. The systematic study of this new geometry leads us to 2-dimensional Lobachevsky geometry (also called non-Euclidean or hyperbolic geometry) which, following the logic of our study, is constructed starting from the properties of its group of motions. Thus in this book we would like to introduce the reader to a theory of geometries which are different from the usual Euclidean geometry of the plane and 3-space, in terms of examples which are accessible to a concrete and intuitive study. The basic method of study is the use of groups of motions, both discrete groups and the groups of motions of geometries. The book does not presuppose on the part of the reader any preliminary knowledge outside the limits of a school geometry course.

This book is a text for junior, senior, or first-year graduate courses traditionally titled Foundations of Geometry and/or Non Euclidean Geometry. The first 29 chapters are for a semester or year course on the foundations of geometry. The remaining chap ters may then be used for either a regular course or independent study courses. Another possibility, which is also especially suited for in-service teachers of high school geometry, is to survey the the fundamentals of absolute geometry (Chapters 1 -20) very quickly and begin earnest study with the theory of parallels and isometries (Chapters 21 -30). The text is self-contained, except that the elementary calculus is assumed for some parts of the material on advanced hyperbolic geometry (Chapters 31 -34). There are over 650 exercises, 30 of which are 10-part true-or-false questions. A rigorous ruler-and-protractor axiomatic development of the Euclidean and hyperbolic planes, including the classification of the isometries of these planes, is balanced by the discussion about this development. Models, such as Taxicab Geometry, are used exten sively to illustrate theory. Historical aspects and alternatives to the selected axioms are prominent. The classical axiom systems of Euclid and Hilbert are discussed, as are axiom systems for three and four-dimensional absolute geometry and Pieri's system based on rigid motions. The text is divided into three parts. The Introduction (Chapters 1 -4) is to be read as quickly as possible and then used for ref erence if necessary.

Hyperbolic Geometry
Complex Analysis
Geometry of Lie Groups
The Arithmetic of Hyperbolic 3-Manifolds
Calculus of One Variable