

## Liboff Quantum Mechanics Solution

**Introductory Quantum Mechanics**Addison Wesley Publishing Company

This book has emerged from an undergraduate course as well as a graduate one, which I have taught for a number of years. Recently, many universities have experimented by bringing quantum theory forward in the curriculum and we follow their example. This book is intended to serve as an introduction to theoretical mechanics and quantum mechanics for chemists. I have included those parts of quantum mechanics which are of greatest fundamental interest and utility, and have developed those parts of classical mechanics which relate to and illuminate them. I try to give a comprehensive treatment wherever possible. The book would acquaint chemists with the quantum structure of the basic object of chemistry, the atom. My intention is to bridge the gap between classical physics, general and inorganic chemistry, and quantum mechanics. For these reasons: 1. I present in one course the basics of theoretical mechanics and quantum mechanics, to emphasise the continuity between them; 2. I have chosen the topics of theoretical mechanics based upon two criteria: a) usefulness for chemical problems: two-body problem; rotational motion of a charged particles (free and in an atom); interaction of a magnetic field with a magnetic dipole; details of small oscillations and oscillations of molecules; b) the need for transition from classical to quantum mechanics: basics of Lagrangian mechanics; basics of Hamiltonian mechanics; 3. I give detailed explanation of an application of the quantum method to simple systems: one-dimensional potential, harmonic oscillator, hydrogen atom, and hydrog- like atoms.

The Physics of Atoms and Quanta is a thorough introduction to experiments and theory in this field. Every classical and modern aspect is covered and discussed in detail. The sixth edition includes new developments, as well as new experiments in quantum entanglement, Schrödingers cat, the quantum computer, quantum information, the atom laser, and much more. A wealth of experiments and problems are included. As this reference ends with the fundamentals of classical bonding, it leads into the authors' more advanced book Molecular Physics and Elements of Quantum Chemistry.

The classic textbook on quantum mechanics from Nobel Prize-winning physicist P. J. E. Peebles This book explains the often counterintuitive physics of quantum mechanics, unlocking this key area of physics for students by enabling them to work through detailed applications of general concepts and ideas. P. J. E. Peebles states general principles first in terms of wave mechanics and then in the standard abstract linear space formalism. He offers a detailed discussion of measurement theory—an essential feature of quantum mechanics—and emphasizes the art of numerical estimates. Along the way, Peebles provides a wealth of physical examples together with numerous problems, some easy, some challenging, but all of them selected because they are physically interesting. Quantum Mechanics is an essential resource for advanced undergraduates and beginning graduate students in physics.

### An Accessible Introduction

**Numerical Analysis and Its Applications**

**Solid State and Quantum Theory for Optoelectronics**

**Introduction to Experiments and Theory ; with ... 173 Problems and Solutions**

**For Atoms, Molecules, Clusters, and Nanocrystals**

**Principles of Quantum Mechanics**

While applications rapidly change one to the next in our commercialized world, fundamental principles behind those applications remain constant. So if one understands those principles well enough and has ample experience in applying them, he or she will be able to develop a capacity for reaching results via conceptual thinking rather than having to This book presents a basic introduction to quantum mechanics. Depending on the choice of topics, it can be used for a one-semester or two-semester course. An attempt has been made to anticipate the conceptual problems students encounter when they first study quantum mechanics. Wherever possible, examples are given to illustrate the underlying physics associated with the mathematical equations of quantum mechanics. To this end, connections are made with corresponding phenomena in classical mechanics and electromagnetism. The problems at the end of each chapter are intended to help students master the course material and to explore more advanced topics. Many calculations exploit the extraordinary capabilities of computer programs such as Mathematica, MatLab, and Maple. Students are urged to use these programs, just as they had been urged to use calculators in the past. The treatment of various topics is rather complete, in that most steps in derivations are included. Several of the chapters go beyond what is traditionally covered in an introductory course. The goal of the presentation is to provide the students with a solid background in quantum mechanics.

This book is a pedagogical presentation of the application of spectral and pseudospectral methods to kinetic theory and quantum mechanics. There are additional applications to astrophysics, engineering, biology and many other fields. The main objective of this book is to provide the basic concepts to enable the use of spectral and pseudospectral methods to solve problems in diverse fields of interest and to a wide audience. While spectral methods are generally based on Fourier Series or Chebychev polynomials, non-classical polynomials and associated quadratures are used for many of the applications presented in the book. Fourier series methods are summarized with a discussion of the resolution of the Gibbs phenomenon. Classical and non-classical quadratures are used for the evaluation of integrals in reaction dynamics including nuclear fusion, radial integrals in density functional theory, in elastic scattering theory and other applications. The subject matter includes the calculation of transport coefficients in gases and other gas dynamical problems based on spectral and pseudospectral solutions of the Boltzmann equation. Radiative transfer in astrophysics and atmospheric science, and applications to space physics are discussed. The relaxation of initial non-equilibrium distributions to equilibrium for several different systems is studied with the Boltzmann and Fokker-Planck equations. The eigenvalue spectra of the linear operators in the Boltzmann, Fokker-Planck and Schrödinger equations are studied with spectral and pseudospectral methods based on non-classical orthogonal polynomials. The numerical methods referred to as the Discrete Ordinate Method, Differential Quadrature, the Quadrature Discretization Method, the Discrete Variable Representation, the Lagrange Mesh Method, and others are discussed and compared. MATLAB codes are provided for most of the numerical results reported in the book - see Link under 'Additional Information' on the the right-hand column.

This book constitutes the refereed proceedings of the First International Workshop on Numerical Analysis and Its Applications, WNAA'96, held in Rousse, Bulgaria, in June 1996. The 57 revised full papers presented were carefully selected and reviewed for inclusion in the volume; also included are 14 invited presentations. All in all, the book offers a wealth of new results and methods of numerical analysis applicable in computational science, particularly in computational physics and chemistry. The volume reflects that the cooperation of computer scientists, mathematicians and scientists provides new numerical tools for computational scientists and, at the same time, stimulates numerical analysis.

Introductory Quantum Mechanics

Physics of Condensed Matter

Nuclear Radiation Interactions

Fundamentals and Model Solutions

Essential Quantum Mechanics

Classical, Quantum, and Relativistic Descriptions

Physics of Condensed Matter is designed for a two-semester graduate course on condensed matter physics for students in physics and materials science. While the book offers fundamental ideas and topic areas of condensed matter physics, it also includes many recent topics of interest on which graduate students may choose to do further research. The text can also be used as a one-semester course for advanced undergraduate majors in physics, materials science, solid state chemistry, and electrical engineering, because it offers a breadth of topics applicable to these majors. The book begins with a clear, coherent picture of simple models of solids and properties and progresses to more advanced properties and topics later in the book. It offers a comprehensive account of the modern topics in condensed matter physics by including introductory accounts of the areas of research in which intense research is underway. The book assumes a working knowledge of quantum mechanics, statistical mechanics, electricity and magnetism and Green's function formalism (for the second-semester curriculum). Covers many advanced topics and recent developments in condensed matter physics which are not included in other texts and are hot areas: Spintronics, Heavy Fermions, Metallic nanoclusters, ZnO, Graphene and graphene-based electronic, Quantum hall effect, High temperature superconductivity, Nanotechnology Offers a diverse number of Experimental techniques clearly simplified Features end of chapter problems

This book introduces mathematicians, physicists, and philosophers to a new, coherent approach to theory and interpretation of quantum physics, in which classical and quantum thinking live peacefully side by side and jointly fertilize the intuition. The formal mathematical core of quantum physics is cleanly separated from the interpretation issues. The book demonstrates that the universe can be rationally and objectively understood from the smallest to the largest levels of modeling. The thermal interpretation featured in this book succeeds without any change in the theory. It involves one radical step, the reinterpretation of an assumption that was virtually never questioned before - the traditional eigenvalue link between theory and observation is replaced by a q-expectation link: Objective properties are given by q-expectations of products of quantum fields and what is computable from these. Averaging over macroscopic spacetime regions produces macroscopic quantities with negligible uncertainty, and leads to classical physics. - Reflects the actual practice of quantum physics. - Models the quantum-classical interface through coherent spaces. - Interprets both quantum mechanics and quantum field theory. - Eliminates probability and measurement from the foundations. - Proposes a novel solution of the measurement problem.

Inspired by Richard Feynman and J.J. Sakurai, A Modern Approach to Quantum Mechanics allows lecturers to expose their undergraduates to Feynman's approach to quantum mechanics while simultaneously giving them a textbook that is well-ordered, logical and pedagogically sound. This book covers all the topics that are typically presented in a standard upper-level course in quantum mechanics, but its teaching approach is new. Rather than organizing his book according to the historical development of the field and jumping into a mathematical discussion of wave mechanics, Townsend begins his book with the quantum mechanics of spin. Thus, the first five chapters of the book succeed in laying out the fundamentals of quantum mechanics with little or no wave mechanics, so the physics is not obscured by mathematics. Starting with spin systems it gives students straightforward examples of the structure of quantum mechanics. When wave mechanics is introduced later, students should perceive it correctly as only one aspect of quantum mechanics and not the core of the subject.

This set of lecture notes on quantum mechanics aims to teach, in a simple and straightforward manner, the basic theory behind the subject, drawing on examples from all fields of physics to provide both background as well as context. The self-contained book includes a review of classical mechanics and some of the necessary mathematics. Both the standard fare of quantum mechanics texts—the harmonic oscillator, the hydrogen atom, angular momentum as well as topics such as symmetry with a discussion on periodic potentials, the relativistic electron, spin and scattering theory are covered. Approximation methods are discussed with a view to applications: these include stationary perturbation theory, the WKB approximation, time dependent perturbations and the variational principle. Together, the seventeen chapters provide a very comprehensive introduction to quantum mechanics. Selected problems are collected at the end of each chapter in addition to the numerous exercises sprinkled throughout the text. The book is written in a simple and elegant style, and is characterized by clarity, depth and excellent pedagogical organization.

Introductory Quantum Mechanics with MATLAB

An Introduction

Technological and Social Perspectives

Quantum Physics

A Reinterpretation of the Tradition

Kinetic Theory

"This book provide a comprehensive coverage of the latest and most relevant knowledge, developments, solutions, and practical applications, related to e-Health, this new field of knowledge able to transform the way we live and deliver services, both from the technological and social perspectives"--Provided by publisher.

This collection of solved problems corresponds to the standard topics covered in established undergraduate and graduate courses in Quantum Mechanics. Problems are also included on topics of interest which are often absent in the existing literature. Solutions are presented in considerable detail, to enable students to follow each step. The emphasis is on stressing the principles and methods used, allowing students to master new ways of thinking and problem-solving techniques. The problems themselves are longer than those usually encountered in textbooks and consist of a number of questions based around a central theme, highlighting properties and concepts of interest. For undergraduate and graduate students, as well as those involved in teaching Quantum Mechanics, the book can be used as a supplementary text or as an independent self-study tool.

"Quantum Chemistry (the branch of Computational Chemistry that applies the laws of Quantum Mechanics to chemical systems) is one of the most dynamic fields of contemporary chemistry, providing a solid foundation for all of chemistry, and serving as the basis for practical, computational methodologies with applications in virtually all branches of chemistry ... The increased sophistication, accuracy and scope of the theory of chemistry are due to a large extent to the spectacular development of quantum chemistry, and in this book the authors have made a remarkable effort to provide a modern account of the field." From the Foreword by Paul Mezey, University of Saskatchewan. Quantum Chemistry: Fundamentals to Applications develops quantum chemistry all the way from the fundamentals, found in Part I, through the applications that make up Part II. The applications include: molecular structure; spectroscopy; thermodynamics; chemical reactions; solvent effects; and excited state chemistry. The importance of this field is underscored by the fact that the 1998 Nobel Prize in Chemistry was awarded for the development of Quantum Chemistry.

The book consists mainly of two parts: Chapter 1 - Chapter 7 and Chapter 8 - Chapter 14. Chapter 1 and Chapter 2 treat design techniques based on linearization of nonlinear systems. An analysis of nonlinear system over quantum mechanics is discussed in Chapter 3. Chapter 4 to Chapter 7 are estimation methods using Kalman filtering while solving nonlinear control systems using iterative approach. Optimal approaches are discussed in Chapter 8 with retarded control of nonlinear system in singular situation, and Chapter 9 extends optimal theory to H-infinity control for a nonlinear control system.Chapters 10 and 11 present the control of nonlinear dynamic systems, twin-rotor helicopter and 3D crane system, which are both underactuated, and dynamic systems. Chapter 12 applies controls to antisynchronization/synchronization in the chaotic models based on Lyapunov exponent theorem, and Chapter 13 discusses developed stability analytic approaches in terms of Lyapunov stability. The analysis of economic activities, especially the relationship between stock return and economic growth, is presented in Chapter 14.

Theory and Applications

Concepts and Applications

Introduction to Quantum Mechanics

Lectures on Quantum Mechanics

Recent Developments

Fundamentals to Applications

This bestselling textbook teaches students how to do quantum mechanics and provides an insightful discussion of what it actually means.

This book is a treatment on the foundational knowledge of Nuclear Science and Engineering. It is an outgrowth of a first-year graduate-level course which the author has taught over the years in the Department of Nuclear Science and Engineering at MIT. The emphasis of the book is on concepts in nuclear science and engineering in contrast to the traditional nuclear physics in a nuclear engineering curriculum. The essential difference lies in the importance we give to the understanding of nuclear radiation and their interactions with matter. We see our students as nuclear engineers who work with all kinds of nuclear devices, from fission and fusion reactors to accelerators and detection systems. In all these complex systems nuclear radiation play a central role. In generating nuclear radiation and using them for beneficial purposes, scientists and engineers must understand the properties of the radiation and how they interact with their surroundings. It is through the control of radiation interactions that we can develop new devices or optimize existing ones to make them more safe, powerful, durable, or economical. This is why radiation interaction is the essence of this book.

Presents a unique approach to grasping the concepts of quantum theory with a focus on atoms, clusters, and crystals Quantum theory of atoms and molecules is vitally important in molecular physics, materials science, nanoscience, solid state physics and many related fields. Introductory Quantum Mechanics with MATLAB is designed to be an accessible guide to quantum theory and its applications. The textbook uses the popular MATLAB programming language for the analytical and numerical solution of quantum mechanical problems, with a particular focus on clusters and assemblies of atoms. The textbook is written by a noted researcher and expert on the topic who introduces density functional theory, variational calculus and other practice-proven methods for the solution of quantum-mechanical problems. This important guide: -Presents the material in a didactical manner to help students grasp the concepts and applications of quantum theory -Covers a wealth of cutting-edge topics such as clusters, nanocrystals, transitions and organic molecules -Offers MATLAB codes to solve real-life quantum mechanical problems Written for master's and PhD students in physics, chemistry, material science, and engineering sciences, Introductory Quantum Mechanics with MATLAB contains an accessible approach to understanding the concepts of quantum theory applied to atoms, clusters, and crystals.

The Second Edition of this concise and compact text offers students a thorough understanding of the basic principles of quantum mechanics and their applications to various physical and chemical problems. This thoroughly class-texted material aims to bridge the gap between the books which give highly theoretical treatments and the ones which present only the descriptive accounts of quantum mechanics. Every effort has been made to make the book explanatory, exhaustive and student friendly. The text focuses its attention on problem-solving to accelerate the student 's grasp of the basic concepts and their applications. What is new to this Edition : Includes new chapters on Field Quantization and Chemical Bonding. Provides new sections on Rayleigh Scattering and Raman Scattering. Offers additional worked examples and problems illustrating the various concepts involved. This textbook is designed as a textbook for postgraduate and advanced undergraduate courses in physics and chemistry. Solutions Manual containing the solutions to chapter-end exercises is available for instructors.

Solution Manual is available for adopting faculty. Click here to request...

First International Workshop, WNAA'96, Rousse, Bulgaria, June 24-26, 1996 Proceedings

Design, Analysis, Estimation and Control

Nonlinear Systems

The Physics of Atoms and Quanta

A Gentle Introduction

Quantum Chemistry

Written primarily for advanced undergraduate and masters level students in physics, this text includes a broad range of topics in applied quantum optics such as laser cooling, Bose-Einstein condensation and quantum information processing.

Balances mathematical discussions with physical discussions. \* Derivations are complete and the theory is applied whenever possible. \* Gasiorowicz is a world class researcher in quantum physics.

A thorough exposition of quantum computing and the underlying concepts of quantum physics, with explanations of the relevant mathematics and numerous examples. The combination of two of the twentieth century's most influential and revolutionary scientific theories, information theory and quantum mechanics, gave rise to a radically new view of computing and information. Quantum information processing explores the implications of using quantum mechanics instead of classical mechanics to model information and its processing. Quantum computing is not about changing the physical substrate on which computation is done from classical to quantum but about changing the notion of computation itself, at the most basic level. The fundamental unit of computation is no longer the bit but the quantum bit or qubit. This comprehensive introduction to the field offers a thorough exposition of quantum computing and the underlying concepts of quantum physics, explaining all the relevant mathematics and offering numerous examples. With its careful development of concepts and thorough explanations, the book makes quantum computing accessible to students and professionals in mathematics, computer science, and engineering. A reader with no prior knowledge of quantum physics but with sufficient knowledge of linear algebra) will be able to gain a fluent understanding by working through the book.

R. Shankar has introduced major additions and updated key presentations in this second edition of Principles of Quantum Mechanics. New features of this innovative text include an entirely rewritten mathematical introduction, a discussion of Time-reversal invariance, and extensive coverage of a variety of path integrals and their applications. Additional highlights include: - Clear, accessible treatment of underlying mathematics - A review of Newtonian, Lagrangian, and Hamiltonian mechanics - Student understanding of quantum theory is enhanced by separate treatment of mathematical theorems and physical postulates - Unsurpassed coverage of path integrals and their relevance in contemporary physics The requisite text for advanced undergraduate- and graduate-level students, Principles of Quantum Mechanics, Second Edition is fully referenced and is supported by many exercises and solutions. The book's self-contained chapters also make it suitable for independent study as well as for courses in applied disciplines.

Theoretical and Quantum Mechanics

A Traditional Approach Emphasizing Connections with Classical Physics

Applications to Kinetic Theory and Quantum Mechanics

Oscillators

Statistical Mechanics

Quantum Mechanics Using Maple ®

The material for these volumes has been selected from the past twenty years' examination questions for graduate students at the University of California at Berkeley, Columbia University, the University of Chicago, MIT, the State University of New York at Buffalo, Princeton University and the University of Wisconsin.

Changes and additions to the new edition of this classic textbook include a new chapter on symmetries, new problems and examples, improved explanations, more numerical problems to be worked on a computer, new applications to solid state physics, and consolidated treatment of time-dependent potentials.

A concise, lucid development of the fundamental structure of quantum mechanics from a thoroughly modern perspective. Focusing on physical and mathematical understanding, with over 60 problems, this compact introduction is invaluable for students and researchers in physics and other fields where quantum mechanics plays an important role

Statistical Mechanics: Fundamentals and Model Solutions, Second Edition Fully updated throughout and with new chapters on the Mayer expansion for classical gases and on cluster expansion for lattice models, this new edition of Statistical Mechanics: Fundamentals and Model Solutions provides a comprehensive introduction to equilibrium statistical mechanics for advanced undergraduate and graduate students of mathematics and physics. The author presents a fresh approach to the subject, setting out the basic assumptions clearly and emphasizing the importance of the thermodynamic limit and the role of convexity. With problems and solutions, the book clearly explains the role of models for physical systems, and discusses and solves various models. An understanding of these models is of increasing importance as they have proved to have applications in many areas of mathematics and physics. Features Updated throughout with new content from the field An established and well-loved textbook Contains new problems and solutions for further learning opportunity Author Professor Teunis C. Dorlas is at the Dublin Institute for Advanced Studies, Ireland.

Handbook of Research on Developments in E-Health and Telemedicine: Technological and Social Perspectives

Second Edition

Spectral Methods in Chemistry and Physics

Fundamentals for Chemists

Quantum Mechanics

A Modern Approach to Quantum Mechanics

This book goes beyond the scope of other works in the field with its thorough treatment of applications in a wide variety of disciplines. The third edition features a new section on constants of motion and symmetry and a new appendix on the Lorentz-Legendre expansion.

An oscillator is dedicated to the generation of signals. It is used in computers, telecoms, watchmaking, astronomy, and metrology. It can be a pendulum, an electronic oscillator based on quartz technology, an optoelectronic oscillator, or an atomic clock, depending on its application. Since wave clocks of antiquity, mechanical clocks invented during the thirteenth century, and the discovery of piezoelectricity by Jacques and Pierre Curie in 1880, oscillators have made great progress. This book does not attempt to tell the story of oscillators, but rather provides an overview of particular oscillator structures through examples from mathematics to oscillators, and from the millimeter scale to the vibration of a building, focusing on recent developments, as we live in a time when technology and mathematical analysis play a vital role.

The lecture notes presented here in facsimile were prepared by Enrico Fermi for students taking his course at the University of Chicago in 1954. They are vivid examples of his unique ability to lecture simply and clearly on the most essential aspects of quantum mechanics. At the close of each lecture, Fermi created a single problem in his students. These challenging exercises were not included in Fermi's notes but were preserved in the notes of his students. This second edition includes a set of these assigned problems as compiled by one of his former students, Robert A. Schluter. Enrico Fermi was awarded the Nobel Prize for Physics in 1938.

Quantum Mechanics Using Maple permits the study of quantum mechanics in a novel, interactive way using the computer algebra and graphics system Maple V. Usually the physics student is distracted from understanding the concepts of modern physics by the need to master unfamiliar mathematics at the same time. In 39 guided Maple sessions the reader explores many standard quantum mechanics problems, as well as some advanced topics that introduce approximation techniques. A solid knowledge of Maple V is acquired as it applies to advanced mathematics relevant for engineering, physics, and applied mathematics. The diskette contains 39 Maple V for Windows worksheet files to reproduce all the problems presented in the text. The suggested exercises can be performed with a minimum of typing.

Quantum Optics

Coherent Quantum Physics

Molecular Quantum Mechanics

Problems and Solutions in Quantum Mechanics

QUANTUM MECHANICS

Problems And Solutions On Quantum Mechanics

Quantum Mechanics: Concepts and Applications provides a clear, balanced and modern introduction to the subject. Written with the student's background and ability in mind the book takes an innovative approach to quantum mechanics by combining the essential elements of the theory with the practical applications: it is therefore both a textbook and a problem solving book in one self-contained volume. Carefully structured, the book starts with the experimental basis of quantum mechanics and then discusses its mathematical tools. Subsequent chapters cover the formal foundations of the subject, the exact solutions of the Schrödinger equation for one and three dimensional potentials, time-independent and time-dependent approximation methods, and finally, the theory of scattering. The text is richly illustrated throughout with many worked examples and numerous problems with step-by-step solutions designed to help the reader master the machinery of quantum mechanics. The new edition has been completely updated and a solutions manual is available on request. Suitable for senior undergraduate courses and graduate courses.

"Quantum Mechanics : An Accessible Introduction brings quantum mechanics to undergraduates in a thorough and uniquely approachable way. Designed from the ground up to address the changing needs of today's students, author Robert Scherrer carefully develops a solid foundation before developing more advanced topics. Introductory chapters explains the historic experimental evidence that motivated the emergence of quantum mechanics, and explain its central role in today's science and technology. Intuitive explanations of a quantum phenomenon provide clear physical motivation for the discussion that follow. Unique Math Interlude chapters ensure that the student has all the mathematical skills required to master quantum mechanics."--Page 4 de la couverture.

The main unique feature of this book is its discussion of Hilbert space and rigged Hilbert space. Suitable for advanced undergraduate students as well as graduate students.

An understanding of quantum mechanics is vital to all students of physics, chemistry and electrical engineering, but requires a lot of mathematical concepts, the details of which are given with great clarity in this book. Various concepts have been derived from first principles, so it can also be used for self-study. The chapters on the JWKB approximation, time-independent perturbation theory and effects of magnetic field stand out for their clarity and easy-to-understand mathematics. Two complete chapters on the linear harmonic oscillator provide a very detailed discussion of one of the most fundamental problems in quantum mechanics. Operator algebra is used to show the ease with which one can calculate the harmonic oscillator wave functions and study the evolution of the coherent state. Similarly, three chapters on angular momentum give a detailed account of this important problem. Perhaps the most attractive feature of the book is the excellent balance between theory and applications and the large number of applications in such diverse areas as astrophysics, nuclear physics, atomic and molecular spectroscopy, solid-state physics, and quantum well structures.

Fundamentals of Modern Physics

Quantum Computing

Notes on Quantum Mechanics

Nonrelativistic Quantum Mechanics

The new edition reflects the progress of physics in both esoteric and pragmatic directions. A complete and detailed presentation, with modern applications, problems, and examples. Annotation copyright Book News, Inc. Portland, Or. One semester introduction to the major concepts of quantum mechanics. Emphasis is on abstract state vectors and on operators.