Sheaves In Geometry And Logic A First Introduction To Topos Theory 2nd Printing Edition

Topos Theory is an important branch of mathematical logic of interest to theoretical computer scientists, logicians and philosophers who study the foundations of mathematics, and to those working in differential geometry and continuum physics. This compendium contains material that was previously available only in specialist journals. This is likely to become the standard reference work for all those interested in the subject.

Categories and sheaves appear almost frequently in contemporary advanced mathematics. This book covers categories, homological algebra and sheaves in a systematic manner starting from scratch and continuing with full proofs to the most recent results in the literature, and sometimes beyond. The authors present the general theory of categories and functors, emphasizing inductive and projective limits, tensor categories, representable functors, ind-objects and localization.

Foundations of Relational Realism presents an intuitive interpretation of quantum mechanics, based on a revised decoherent histories interpretation, structured within a category theoretic topological formalism.

This text exposes the basic features of cohomology of sheaves and its applications. The general theory of sheaves is very limited and no essential result is obtainable without turn ing to particular classes of topological spaces. The most satis factory general class is that of locally compact spaces and it is the study of such spaces which occupies the central part of this text. The fundamental concepts in the study of locally compact spaces is cohomology with compact support and a particular class of sheaves,the so-called soft sheaves. This class plays a double role as the basic vehicle for the internal theory and is the key to applications in analysis. The basic example of a soft sheaf is the sheaf of smooth functions on ~n or more generally on any smooth manifold. A rather large effort has been made to demon strate the relevance of sheaf theory in even the most elementary analysis. This process has been reversed in order to base the fundamental calculations in sheaf theory on elementary analysis.

Theories, Sites, Toposes

Homology

Cohomology of Sheaves

Global Calculus

The Categorial Analysis of Logic

Foundations of Relational Realism

New corrected printing of a well-established text on logic at the introductory level.

This is the first book on category theory for a broad philosophical readership. There is no other discussion of category theory comparable in its scope. It is designed to show the interest and significant of category theory for philosophers working in a range of areas, including mathematics, proof theory, computer science, ontology, physics, biology, cognition, mathematical modelling, the structure of scientific theories, and the structure of the world. Moreover, it does this in a way that is accessible to non specialists. Each chapter is written by either a category-theorist or a philosopher working in one of the represented fields, in a way that builds on the concepts already familiar to philosophers working in these areas. The book is split into two halves. The 'pure' chapters focus on the use of category theory for mathematical, foundational, and logical purposes, while the 'applied' chapters consider the use of category theory for representational purposes, investigating category theory as a framework for theories of physics and biology, for mathematical modelling more generally, and for the structure of scientific theories. Book jacket.

In presenting this treatment of homological algebra, it is a pleasure to acknowledge the help and encouragement which I have had from all sides. Homological algebra arose from many sources in algebra and topology. Decisive examples came from the study of group extensions and their factor sets, a subject I learned in joint work with OTTO SCHIL LING. A further development of homological ideas, with a view to their topological applications, came in my long collaboration with SAMUEL ElLENBERG; to both collaborators, especial thanks. For many years the Air Force Office of Scientific Research supported my research projects on various subjects now summarized here; it is a pleasure to acknowledge their lively understanding of basic science. Both REINHOLD BAER and JOSEF SCHMID read and commented on my entire manuscript; their advice has led to many improvements. ANDERS KOCK and JACQUES RIGUET have read the entire galley proof and caught many slips and obscurities. Among the others whose sug gestions have served me well, I note FRANK ADAMS, LOUIS AUSLANDER, WILFRED COCKCROFT, ALBRECHT DOLD, GEOFFREY HORROCKS, FRIED RICH KASCH, JOHANN LEICHT, ARUNAS LIULEVICIUS, JOHN MOORE, DIE TER PUPPE, JOSEPH YAO, and a number of my current students at the University of Chicago - not to m~ntion the auditors of my lectures at Chicago, Heidelberg, Bonn, Frankfurt, and Aarhus. My wife, DOROTHY, has cheerfully typed more versions of more chapters than she would like to count. Messrs.

Category Theory has developed rapidly. This book aims to present those ideas and methods which can now be effectively used by Mathe maticians working in a variety of other fields of Mathematical research. This occurs at several levels. On the first level, categories provide a convenient conceptual language, based on the notions of category, functor, natural transformation, contravariance, and functor category. These notions are presented, with appropriate examples, in Chapters I and II. Next comes the fundamental idea of an adjoint pair of functors. This appears in many substantially equivalent forms: That of universal construction, that of direct and inverse limit, and that of pairs offunctors with a natural isomorphism between corresponding sets of arrows. All these forms, with their interrelations, are examined in Chapters III to V. The slogan is "Adjoint functors arise everywhere". Alternatively, the fundamental notion of category theory is that of a monoid -a set with a binary

operation of multiplication which is associative and which has a unit; a category itself can be regarded as a sort of general ized monoid. Chapters VI and VII explore this notion and its generaliza tions. Its close connection to pairs of adjoint functors illuminates the ideas of universal algebra and culminates in Beck's theorem characterizing categories of algebras; on the other hand, categories with a monoidal structure (given by a tensor product) lead inter alia to the study of more convenient categories of topological spaces.

A Concise Course in Algebraic Topology

Sketches of an Elephant: A Topos Theory Compendium

An Introduction

Vertex Algebras and Algebraic Curves: Second Edition

Algebraic Geometry over the Complex Numbers

Cartan's Generalization of Klein's Erlangen Program

Noncommutative geometry, inspired by quantum physics, describes singular spaces by their noncommutative coordinate algebras and metric structures by Dirac-like operators. Such metric geometries are described mathematically by Connes' theory of spectral triples. These lectures, delivered at an EMS Summer School on noncommutative geometry and its applications, provide an overview of spectral triples based on examples. This introduction is aimed at graduate students of both mathematics and theoretical physics. It deals with Dirac operators on spin manifolds, noncommutative tori, Moyal quantization and tangent groupoids, action functionals, and isospectral deformations. The structural framework is the concept of a noncommutative spin geometry; the conditions on spectral triples which determine this concept are developed in detail. The emphasis throughout is on gaining understanding by computing the details of specific examples. The book provides a middle ground between a comprehensive text and a narrowly focused research monograph. It is intended for self-study, enabling the reader to gain access to the essentials of noncommutative geometry. New features since the original course are an expanded bibliography and a survey of more recent examples and applications of spectral triples.

The first of its kind, this book presents a widely accessible exposition of topos theory, aimed at the philosopher-logician as well as the mathematician. It is suitable for individual study or use in class at the graduate level (it includes 500 exercises). It begins with a fully motivated introduction to category theory itself, moving always from the particular example to the abstract concept. It then introduces the notion of elementary topos, with a wide range of examples and goes on to develop its theory in depth, and to elicit in detail its relationship to Kripke's intuitionistic semantics, models of classical set theory and the conceptual framework of sheaf theory (l̀ocalization" of truth). Of particular interest is a Dedekind-cuts style construction of number systems in topoi, leading to a model of the intuitionistic continuum in which a D̀edekind-real" becomes represented as a c̀ontinuously-variable classical real number". The second edition contains a new chapter, entitled Logical Geometry, which introduces the reader to the theory of geometric morphisms of Grothendieck topoi, and its model-theoretic rendering by Makkai and Reyes. The aim of this chapter is to explain why Deligne's theorem about the existence of points of coherent topoi is equivalent to the classical Completeness theorem for g̀eometric" first-order formulae.

Algebraic topology is a basic part of modern mathematics, and some knowledge of this area is indispensable for any advanced work relating to geometry, including topology itself, differential geometry, algebraic geometry, and Lie groups. This book provides a detailed treatment of algebraic topology both for teachers of the subject and for advanced graduate students in mathematics either specializing in this area or continuing on to other fields. J. Peter May's approach reflects the enormous internal developments within algebraic topology over the past several decades, most of which are largely unknown to mathematicians in other fields. But he also retains the classical presentations of various topics where appropriate. Most chapters end with problems that further explore and refine the concepts presented. The final four chapters provide sketches of substantial areas of algebraic topology that are normally omitted from introductory texts, and the book concludes with a list of suggested readings for those interested in delving further into the field.

This book records my efforts over the past four years to capture in words a description of the form and function of Mathematics, as a background for the Philosophy of Mathematics. My efforts have been encouraged by lec tures that I have given at Heidelberg under the auspices of the Alexander von Humboldt Stiftung, at the University of Chicago, and at the University of Minnesota, the latter under the auspices of the Institute for Mathematics and Its Applications. Jean Benabou has carefully read the entire manuscript and has offered incisive comments. George Glauberman, Car los Kenig, Christopher Mulvey, R. Narasimhan, and Dieter Puppe have provided similar comments on chosen chapters. Fred Linton has pointed out places requiring a more exact choice of wording. Many conversations with George Mackey have given me important insights on the nature of Mathematics. I have had similar help from Alfred Aeppli, John Gray, Jay Goldman, Peter Johnstone, Bill Lawvere, and Roger Lyndon. Over the years, I have profited from discussions of general issues with my colleagues Felix Browder and Melvin Rothenberg. Ideas from Tammo Tom Dieck, Albrecht Dold, Richard Lashof, and Ib Madsen have assisted in my study of geometry. Jerry Bona and B.L. Foster have helped with my examina tion of mechanics. My observations about logic have been subject to con structive scrutiny by Gert Miiller, Marian Boykan Pour-El, Ted Slaman, R. Voreadou, Volker Weispfennig, and Hugh Woodin.

Special Topics in Order, Topology, Algebra, and Sheaf Theory

Topoi

Glen E. Bredon

Practical Foundations of Mathematics

Volume 2

An Introduction to Noncommutative Geometry

In the last five decades various attempts to formulate theories of quantum gravity have been made, but none has fully succeeded in becoming the quantum theory of gravity. One possible explanation for this failure might be the unresolved fundamental issues in quantum theory as it stands now. Indeed, most approaches to quantum gravity adopt standard quantum theory as their starting point, with the hope that the theory ' s unresolved issues will get solved along the way. However, these fundamental issues may need to be solved before attempting to define a quantum theory of gravity. The present text adopts this point of view, addressing the following basic questions: What are the main conceptual issues in quantum theory? How can these issues be solved within a new theoretical framework of quantum theory? A possible way to overcome critical issues in present-day quantum physics – such as a priori assumptions about space and time that are not compatible with a theory of quantum gravity, and the impossibility of talking about systems without reference to an external observer – is through a reformulation of quantum theory in terms of a different mathematical framework called topos theory. This course-tested primer sets out to explain to graduate students and newcomers to the field alike, the reasons for choosing topos theory to resolve the above-mentioned issues and how it brings quantum physics back to looking more like a " neo-realist " classical physics theory again.

This book offers a new algebraic approach to set theory. The authors introduce a particular kind of algebra, the Zermelo-Fraenkel algebras, which arise from the familiar axioms of Zermelo-Fraenkel set theory. Furthermore, the authors explicitly construct these

algebras using the theory of bisimulations. Their approach is completely constructive, and contains both intuitionistic set theory and topos theory. In particular it provides a uniform description of various constructions of the cumulative hierarchy of sets in forcing models, sheaf models and realizability models. Graduate students and researchers in mathematical logic, category theory and computer science should find this book of great interest, and it should be accessible to anyone with a background in categorical logic.
This two-volume monograph obtains fundamental notions and results of the standard differential geometry of smooth (CINFINITY) manifolds, without using differential calculus. Here, the sheaf-theoretic character is emphasised. This has theoretical advantages such as greater perspective, clarity and unification, but also practical benefits ranging from elementary particle physics, via gauge theories and theoretical cosmology (`differential spaces'), to non-linear PDEs (generalised functions). Thus, more general applications, which are no longer `smooth' in the classical sense, can be coped with. The treatise might also be construed as a new systematic endeavour to confront the ever-increasing notion that the `world around us is far from being smooth enough'. Audience: This work is intended for postgraduate students and researchers whose work involves differential geometry, global analysis, analysis on manifolds, algebraic topology, sheaf theory, cohomology, functional analysis or abstract harmonic analysis.
An introduction to the theory of toposes which begins with illustrative examples and goes on to explain the underlying ideas of topology and sheaf theory as well as the general theory of elementary toposes and geometric morphisms and their relation to logic.
A First Introduction to Topos Theory
Foundations of Algebraic Geometry. --; 29
Elementary Categories, Elementary Toposes
Logic and Structure
Complex Geometry
The Prospect of a New Logic for Philosophy
This book is about the basis of mathematical reasoning both in pure mathematics itself (particularly algebra and topology) and in computer science (how and what it means to prove correctness of programs). It contains original material and original developments of standard material, so it is also for professional researchers, but as it deliberately transcends disciplinary boundaries and challenges many established attitudes to the foundations of mathematics, the reader is expected to be open minded about these things.
Easily accessible Includes recent developments Assumes very little knowledge of differentiable manifolds and functional analysis Particular emphasis on topics related to mirror symmetry (SUSY, Kaehler-Einstein metrics, Tian-Todorov lemma)
Focusing on topos theory's integration of geometric and logical ideas into the foundations of mathematics and theoretical computer science, this volume explores internal category theory, topologies and sheaves, geometric morphisms, and other subjects. 1977 edition.
Cartan geometries were the first examples of connections on a principal bundle. They seem to be almost unknown these days, in spite of the great beauty and conceptual power they confer on geometry. The aim of the present book is to fill the gap in the literature on differential geometry by the missing notion of Cartan connections. Although the author had in mind a book accessible to graduate students, potential readers would also include working differential geometers who would like to know more about what Cartan did, which was to give a notion of "espaces généralisés" (= Cartan geometries) generalizing homogeneous spaces (= Klein geometries) in the same way that Riemannian geometry generalizes Euclidean geometry. In addition, physicists will be interested to see the fully satisfying way in which their gauge theory can be truly regarded as geometry.
First Order Categorical Logic
Geometry
Etale Homotopy
Sets, Models and Proofs
Categories for the Working Philosopher
Handbook of Categorical Algebra: Volume 3, Sheaf Theory
This is a relatively fast paced graduate level introduction to complex algebraic geometry, from the basics to the frontier of the subject. It covers sheaf theory, cohomology, some Hodge theory, as well as some of the more algebraic aspects of algebraic geometry. The author frequently refers the reader if the treatment of a certain topic is readily available elsewhere but goes into considerable detail on topics for which his treatment puts a twist or a more transparent viewpoint. His cases of exploration and are chosen very carefully and deliberately. The textbook achieves its purpose of taking new students of complex algebraic geometry through this a deep yet broad introduction to a vast subject, eventually bringing them to the forefront of the topic via a non-intimidating style.
The power that analysis, topology and algebra bring to geometry has revolutionised the way geometers and physicists look at conceptual problems. Some of the key ingredients in this interplay are sheaves, cohomology, Lie groups, connections and differential operators. In Global Calculus, the appropriate formalism for these topics is laid out with numerous examples and applications by one of the experts in differential and algebraic geometry. Ramanan has chosen an uncommon but natural path through the subject. In this almost completely self-contained account, these topics are developed from scratch. The basics of Fourier transforms, Sobolev theory and interior regularity are proved at the same time as symbol calculus, culminating in beautiful results in global analysis, real and complex. Many new perspectives on traditional and modern questions of differential analysis and geometry are the hallmarks of the book. The book is suitable for a first year graduate course on Global Analysis.
Vertex algebras are algebraic objects that encapsulate the concept of operator product expansion from two-dimensional conformal field theory. Vertex algebras are fast becoming ubiquitous in many areas of modern mathematics, with applications to representation theory, algebraic geometry, the theory of finite groups, modular functions, topology, integrable systems, and combinatorics. This book is an introduction to the theory of vertex algebras with a particular emphasis on the relationship with the geometry of algebraic curves. The notion of a vertex algebra is introduced in a coordinate-independent way, so that vertex operators become well defined on arbitrary smooth algebraic curves, possibly equipped with additional data, such as a vector bundle. Vertex algebras then appear as the algebraic objects encoding the geometric structure of various moduli spaces associated with algebraic curves. Therefore they may be used to give a geometric interpretation of various questions of representation theory. The book contains many original results, introduces important new concepts, and brings new insights into the theory of vertex algebras. The authors have made a great effort to make the book self-contained and accessible

to readers of all backgrounds. Reviewers of the first edition anticipated that it would have a long-lasting influence on this exciting field of mathematics and would be very useful for graduate students and researchers interested in the subject. This second edition, substantially improved and expanded, includes several new topics, in particular an introduction to the Beilinson-Drinfeld theory of factorization algebras and the geometric Langlands correspondence.

Sheaves in Geometry and LogicA First Introduction to Topos TheorySpringer Science & Business Media

Categories and Sheaves

Topos Theory

Categories for the Working Mathematician

Categorical Foundations

A Topological Approach to Quantum Mechanics and the Philosophy of Nature

Stone Spaces

A graduate-level textbook that presents basic topology from the perspective of category theory. This graduate-level textbook on topology takes a unique approach: it reintroduces basic, point-set topology from a more modern, categorical perspective. Many graduate students are familiar with the ideas of point-set topology and they are ready to learn something new about them. Teaching the subject using category theory—a contemporary branch of mathematics that provides a way to represent abstract concepts—both deepens students' understanding of elementary topology and lays a solid foundation for future work in advanced topics. After presenting the basics of both category theory and topology, the book covers the universal properties of familiar constructions and three main topological properties—connectedness, Hausdorff, and compactness. It presents a fine-grained approach to convergence of sequences and filters; explores categorical limits and colimits, with examples; looks in detail at adjunctions in topology, particularly in mapping spaces; and examines additional adjunctions, presenting ideas from homotopy theory, the fundamental groupoid, and the Seifert van Kampen theorem. End-of-chapter exercises allow students to apply what they have learned. The book expertly guides students of topology through the important transition from undergraduate student with a solid background in analysis or point-set topology to graduate student preparing to work on contemporary problems in mathematics.

Publisher Description

The aim of this book is to construct categories of spaces which contain all the C?-manifolds, but in addition infinitesimal spaces and arbitrary function spaces. To this end, the techniques of Grothendieck toposes (and the logic inherent to them) are explained at a leisurely pace and applied. By discussing topics such as integration, cohomology and vector bundles in the new context, the adequacy of these new spaces for analysis and geometry will be illustrated and the connection to the classical approach to C?-manifolds will be explained.

At last: geometry in an exemplary, accessible and attractive form! The authors emphasise both the intellectually stimulating parts of geometry and routine arguments or computations in concrete or classical cases, as well as practical and physical applications. They also show students the fundamental concepts and the difference between important results and minor technical routines. Altogether, the text presents a coherent high school curriculum for the geometry course, naturally backed by numerous examples and exercises.

Applications of Sheaves

Mathematics Form and Function

Sheaves in Geometry and Logic

A First Course in Topos Quantum Theory

Topology

This textbook provides a concise and self-contained introduction to mathematical logic, with a focus on the fundamental topics in first-order logic and model theory. Including examples from several areas of mathematics (algebra, linear algebra and analysis), the book illustrates the relevance and usefulness of logic in the study of these subject areas. The authors start with an exposition of set theory and the axiom of choice as used in everyday mathematics. Proceeding at a gentle pace, they go on to present some of the first important results in model theory, followed by a careful exposition of Gentzen-style natural deduction and a detailed proof of Gödel's completeness theorem for first-order logic. The book then explores the formal axiom system of Zermelo and Fraenkel before concluding with an extensive list of suggestions for further study. The present volume is primarily aimed at mathematics students who are already familiar with basic analysis, algebra and linear algebra. It contains numerous exercises of varying difficulty and can be used for self-study, though it is ideally suited as a text for a one-semester university course in the second or third year.

Sheaves arose in geometry as coefficients for cohomology and as descriptions of the functions appropriate to various kinds of manifolds. Sheaves also appear in logic as carriers for models of set theory. This text presents topos theory as it has developed from the study of sheaves. Beginning with several examples, it explains the underlying ideas of topology and sheaf theory as well as the general theory of elementary toposes and geometric morphisms and their relation to logic.

A unified treatment of the corpus of mathematics that has developed out of M. H. Stone's representation theorem for Boolean algebras (1936) which has applications in almost every area of modern mathematics.

This book introduces a set of methods and techniques for studying mathematical theories and relating them to each other through the use of Grothendieck toposes.

Models for Smooth Infinitesimal Analysis

Model-Theoretical Methods in the Theory of Topoi and Related Categories

Geometry of Vector Sheaves

Sheaf Theory

An Axiomatic Approach to Differential Geometry Volume II: Geometry. Examples and Applications

A High School Course

This work has been selected by scholars as being culturally important and is part of the knowledge base of civilization as we know it. This work is in the public domain in the United States of America, and possibly other nations. Within the United States, you may freely *copy and distribute this work, as no entity (individual or corporate) has a copyright on the body of the work. Scholars believe, and we concur, that this work is important enough to be preserved, reproduced, and made generally available to the public. To ensure a quality reading experience, this work has been proofread and republished using a format that seamlessly blends the original graphical elements with text in an easy-to-read typeface. We appreciate your support of the preservation process, and thank you for being an important part of keeping this knowledge alive and relevant.*

Third in a three part set, this volume introduces topos theory and the idea of sheaves.

The book covers elementary aspects of category theory and topos theory. It has few mathematical prerequisites, and uses categorical methods throughout rather than beginning with set theoretic foundations. It works with key notions such as cartesian closedness, adjunctions, regular categories, and the internal logic of a topos. Full statements and elementary proofs are given for the central theorems, including the fundamental theorem of toposes, the sheafification theorem, and the construction of Grothendieck toposes over any topos as base. Three chapters discuss applications of toposes in detail, namely to sets, to basic differential geometry, and to recursive analysis. - ;Introduction; PART I: CATEGORIES: Rudimentary structures in a category; Products, equalizers, and their duals; Groups; Sub-objects, pullbacks, and limits; Relations; Cartesian closed categories; Product operators and others; PART II: THE CATEGORY OF CATEGORIES: Functors and categories; Natural transformations; Adjunctions; Slice categories; Mathematical foundations; PART III: TOPOSES: Basics; The internal language; A soundness proof for topos logic; From the internal language to the topos; The fundamental theorem; External semantics; Natural number objects; Categories in a topos; Topologies; PART IV: SOME TOPOSES: Sets; Synthetic differential geometry; The effective topos; Relations in regular categories; Further reading; Bibliography; Index. -

"The old logic put thought in fetters, while the new logic gives it wings." For the past century, philosophers working in the tradition of Bertrand Russell - who promised to revolutionise philosophy by introducing the 'new logic' of Frege and Peano - have employed predicate logic as their formal language of choice. In this book, Dr David Corfield presents a comparable revolution with a newly emerging logic - modal homotopy type theory. Homotopy type theory has recently been developed as a new foundational language for mathematics, with a strong philosophical pedigree. Modal Homotopy Type Theory: The Prospect of a New Logic for Philosophy offers an introduction to this new language and its modal extension, illustrated through innovative applications of the calculus to language, metaphysics, and mathematics. The chapters build up to the full language in stages, right up to the application of modal homotopy type theory to current geometry. From a discussion of the distinction between objects and events, the intrinsic treatment of structure, the conception of modality as a form of general variation to the representation of constructions in modern geometry, we see how varied the applications of this powerful new language can be.

Relating and Studying Mathematical Theories Through Topos-theoretic 'bridges'

A Categorical Approach

Modal Homotopy Type Theory

Algebraic Set Theory

Differential Geometry

Proceedings of the Research Symposium on Applications of Sheaf Theory to Logic, Algebra and Analysis, Durham, July 9-21, 1977