# *The Foundations Of Arithmetic A Logico Mathematical Enquiry Into Concept Number Gottlob Frege*

**This collection of papers from various areas of mathematical logic showcases the remarkable breadth and richness of the field. Leading authors reveal how contemporary technical results touch upon foundational questions about the nature of mathematics. Highlights of the volume include: a history of Tennenbaum's theorem in arithmetic; a number of papers on Tennenbaum phenomena in weak arithmetics as well as on other aspects of arithmetics, such as interpretability; the transcript of Gödel's previously unpublished 1972–1975 conversations with Sue Toledo, along with an appreciation of the same by Curtis Franks; Hugh Woodin's paper arguing against the generic multiverse view; Anne Troelstra's history of intuitionism through 1991; and Aki Kanamori's history of the Suslin problem in set theory. The book provides a historical and philosophical treatment of particular theorems in arithmetic and set theory, and is ideal for researchers and graduate students in mathematical logic and philosophy of mathematics.**

**The transition from school mathematics to university mathematics is seldom straightforward. Students are faced with a disconnect between the algorithmic and informal attitude to mathematics at school, versus a new emphasis on proof, based on logic, and a more abstract development of general concepts, based on set theory. The authors have many years' experience of the potential difficulties involved, through teaching first-year undergraduates and researching the ways in which students and mathematicians think. The book explains the motivation behind abstract foundational material based on students' experiences of school mathematics, and explicitly suggests ways students can make sense of formal ideas. This second edition takes a significant step forward by not only making the transition from intuitive to formal methods, but also by reversing the process- using structure theorems to prove that formal systems have visual and symbolic interpretations that enhance mathematical thinking. This is exemplified by a new chapter on the theory of groups. While the first edition extended counting to infinite cardinal numbers, the second also extends the real numbers rigorously to larger ordered fields. This links intuitive ideas in calculus to the formal epsilon-delta methods of analysis. The approach here is not the conventional one of 'nonstandard analysis', but a simpler, graphically based treatment which makes the notion of an infinitesimal natural and straightforward. This allows a further vision of the wider world of mathematical thinking in which formal definitions and proof lead to amazing new ways of defining, proving, visualising and symbolising mathematics beyond previous expectations.**

**The volume is the first collection of essays that focuses on Gottlob Frege's Basic Laws of Arithmetic (1893/1903), highlighting both the technical and the philosophical richness of Frege's magnum opus. It brings together twenty-two renowned Frege scholars whose contributions discuss a wide range of topics arising from both volumes of Basic Laws of Arithmetic. The original chapters in this volume make vivid the importance and originality of Frege's masterpiece, not just for Frege scholars but for the study of the history of logic, mathematics, and philosophy.**

**The Search for Certainty : A Philosophical Account of Foundations of Mathematics**

**A Study in the Philosophy of Science**

**An Adventurer's Guide to Number Theory**

**A Logico-mathematical Enquiry Into the Concept of Number. English Translation by J.L. Austin**

**David Hilbert's Lectures on the Foundations of Arithmetic and Logic 1917-1933**

**A Logical-mathematical Investigation Into the Concept of Number 1884**

Part of theLongman Library of Primary Sources in Philosophy, this edition of Frege's Foundations of Arithmetic is framed by a pedagogical structure designed to make this important work of philosophy more accessible and meaningful for undergraduates.

The Foundations of ArithmeticA Logico-Mathematical Enquiry Into the Concept of NumberNorthwestern University Press

What is mathematics about? Does the subject-matter of mathematics exist independently of the mind or are they mental constructions? How do we know mathematics? Is mathematical knowledge logical knowledge? And how is mathematics applied to the material world? In this introduction to the philosophy of mathematics, Michele Friend examines these and other ontological and epistemological problems raised by the content and practice of mathematics. Aimed at a readership with limited proficiency in mathematics but with some experience of formal logic it seeks to strike a balance between conceptual accessibility and correct representation of the issues. Friend examines the standard theories of mathematics - Platonism, realism, logicism, formalism, constructivism and structuralism - as well as some less standard theories such as psychologism, fictionalism and Meinongian philosophy of mathematics. In each case Friend explains what characterises the position and where the divisions between them lie, including some of the arguments in favour and against each. This book also explores particular questions that occupy present-day philosophers and mathematicians such as the problem of infinity, mathematical intuition and the relationship, if any, between the philosophy of mathematics and the practice of mathematics. Taking in the canonical ideas of Aristotle, Kant, Frege and Whitehead and Russell as well as the challenging and innovative work of recent philosophers like Benacerraf, Hellman, Maddy and Shapiro, Friend provides a balanced and accessible introduction suitable for upper-level undergraduate courses and the non-specialist.

Philosophy and Foundations of Mathematics

A Philosophical Account of Foundations of Mathematics

Dedicated to A. A. Fraenkel on His Seventieth Anniversary

Wittgenstein, Finitism, and the Foundations of Mathematics

Number Systems and the Foundations of Analysis

The Foundations of Arithmetic

The core of Volume 3 consists of lecture notes for seven sets of lectures Hilbert gave (often in collaboration with Bernays) on the foundations of mathematics between 1917 and 1926. These texts make possible for the first time a detailed reconstruction of the rapid development of Hilbert's foundational thought during this period, and show the increasing dominance of the metamathematical perspective in his logical work: the emergence of modern mathematical logic; the explicit raising of questions of completeness, consistency and decidability for logical systems; the investigation of the relative strengths of various logical calculi; the birth and evolution of proof theory, and the parallel emergence of Hilbert’s finitist standpoint. The lecture notes are accompanied by numerous supplementary documents, both published and unpublished, including a complete version of Bernays’s Habilitationschrift of 1918, the text of the first edition of Hilbert and Ackermann’s Grundzüge der theoretischen Logik (1928), and several shorter lectures by Hilbert from the later 1920s. These documents, which provide the background to Hilbert and Bernays’s monumental Grundlagen der Mathematik (1934, 1938), are essential for understanding the development of modern mathematical logic, and for reconstructing the interactions between Hilbert, Bernays, Brouwer, and Weyl in the philosophy of mathematics.

Moritz Pasch (1843-1930) is justly celebrated as a key figure in the history of axiomatic geometry. Less well known are his contributions to other areas of foundational research. This volume features English translations of 14 papers Pasch published in the decade 1917-1926. In them, Pasch argues that geometry and, more surprisingly, number theory are branches of empirical science; he provides axioms for the combinatorial reasoning essential to Hilbert’s program of consistency proofs; he explores "implicit definition" (a generalization of definition by abstraction) and indicates how this technique yields an "empiricist" reconstruction of set theory; he argues that we cannot fully understand the logical structure of mathematics without clearly distinguishing between decidable and undecidable properties; he offers a rare glimpse into the mind of a master of axiomatics, surveying in detail the thought experiments he employed as he struggled to identify fundamental mathematical principles; and much more. This volume will: Give English speakers access to an important body of work from a turbulent and pivotal period in the history of mathematics, help us look beyond the familiar triad of formalism, intuitionism, and logicism, show how deeply we can see with the help of a guide determined to present fundamental mathematical ideas in ways that match our human capacities, will be of interest to graduate students and researchers in logic and the foundations of mathematics.

This is the first complete English translation of Gottlob Frege's Grundgesetze der Arithmetik (originally published in two volumes, 1893 and 1903), with introduction and annotation. The importance of Frege's ideas within contemporary philosophy would be hard to exaggerate. He was, to allintents and purposes, the inventor of mathematical logic, and the influence exerted on modern philosophy of language and logic, and indeed on general epistemology, by the philosophical framework within which his technical contributions were conceived and developed has been so deep that he has astrong case to be regarded as the inventor of much of the agenda of modern analytical philosophy itself. Two of Frege's three principal books - the Begriffsschrift (1879) and Grundlagen der Arithmetik (1884) - have been available in English translation for many years, as have all the most important of his other, article-length writings. Grundgesetze was to have been the summit of Frege's life's work -a rigorous demonstration of how the fundamental laws of the classical pure mathematics of the natural and real numbers could be derived from principles which, in his view, were purely logical. A letter received from Bertrand Russell shortly before the publication of the second volume made Fregerealise that Axiom V of his system, governing identity for value-ranges, led to contradiction. But much of the main thrust of Frege's project can be salvaged. The continuing importance of the Grundgesetze lies not only in its bearing on issues in the foundations of mathematics but in its model of philosophical inquiry. Frege's ability to locate the essential questions, his integration of logical and philosophical analysis, and his rigorous approach tocriticism and argument in general are vividly in evidence in this, his most ambitious work.

Building the Foundation: Whole Numbers in the Primary Grades
Making Numbers
Frege, Dedekind, and Peano on the Foundations of Arithmetic
Essays on the Foundations of Mathematics by Moritz Pasch
Introducing Philosophy of Mathematics
A Logico-Mathematical Enquiry Into the Concept of Number

**Mathematical logic grew out of philosophical questions regarding the foundations of mathematics, but logic has now outgrown its philosophical roots, and has become an integral part of mathematics in general. This book is designed for students who plan to specialize in logic, as well as for those who are interested in the applications of logic to other areas of mathematics. Used as a text, it could form the basis of a beginning graduate-level course. There are three main chapters: Set Theory, Model Theory, and Recursion Theory. The Set Theory chapter describes the set-theoretic foundations of all of mathematics, based on the ZFC axioms. It also covers technical results about the Axiom of Choice, well-orderings, and the theory of uncountable cardinals. The Model Theory chapter discusses predicate logic and formal proofs, and covers the Completeness, Compactness, and Lowenheim-Skolem Theorems, elementary submodels, model completeness, and applications to algebra. This chapter also continues the foundational issues begun in the set theory chapter. Mathematics can now be viewed as formal proofs from ZFC. Also, model theory leads to models of set theory. This includes a discussion of absoluteness, and an analysis of models such as H( ) and R( ). The Recursion Theory chapter develops some basic facts about computable functions, and uses them to prove a number of results of foundational importance; in particular, Church's theorem on the undecidability of logical consequence, the incompleteness theorems of Godel, and Tarski's theorem on the non-definability of truth.**

**This witty introduction to number theory deals with the properties of numbers and numbers as abstract concepts. Topics include primes, divisibility, quadratic forms, and related theorems.**

**"Although almost unknown in his lifetime, it was Gottlob Frege (1848-1925) who set the agenda for much of twentieth-century philosophy." "His 'concept script' overthrew Aristotle's long-established system of logic and underlies all subsequent developments in the subject. His radically new approach to the foundations of arithmetic, based on fresh definitions of the terms 'zero', 'one' and 'successor', revolutionized our understanding of mathematics. And his important insights into the nature of language and meaning provided the framework for Russell, Wittgenstein and twentieth-century linguistic analysis.**

**In this superb survey of his evolving ideas, Anthony Kenny explains and assesses the full range of Frege's work and reveals why it still forms an ideal introduction to modern analytic philosophy. Even after seventy years, he concludes, Frege remains an absolutely central figure, one of those rare thinkers who wrote 'prose which is accessible and attractive on first acquaintance and yet which repays rereading over a lifetime'."--BOOK JACKET.Title Summary field provided by Blackwell North America, Inc. All Rights Reserved**

**Frege, Dedekind, and Peano on the Foundations of Arithmetic (Routledge Revivals)**

**Bemerkungen Über Die Grundlagen Der Mathematik**

**The Foundations of Arithmetic ... Translation by J.L. Austin ... Second Revised Edition. Ger. & Eng**

**An Introduction to the Founder of Modern Analytic Philosophy**

**Foundations of Arithmetic Differential Geometry**

**Set Theory, Arithmetic, and Foundations of Mathematics**

*Part of the Longman Library of Primary Sources in Philosophy," this edition ofFrege's Foundations of Arithmetic is framed by a pedagogical structure designed to make this important work of philosophy more accessible and meaningful for readers. A General Introduction includes the work's historical context, a discussion of historical influences, and biographical information on Gottlob Frege. The conclusion discusses how the work has influenced other philosophers and why it is important today. Annotations and notes from the editor clarify difficult passages for greater understanding, and a bibliography gives the reader additional resources for further study.*

*Geared toward upper-level undergraduates and graduate students, this treatment examines the basic paradoxes and history of set theory and advanced topics such as relations and functions, equipollence, more. 1960 edition.*

*This is a careful, historically informed study of Wittgenstein's philosophy of mathematics, tracing the work development of his thinking from the 1920s through to the 1950s, in the context of the mathematical and philosophical work of the times.*

*Essays on the Foundations of Mathematics*

*L. E. J. Brouwer*

*Selected Readings*

*The Foundations of Mathematics*

*Theorems, Philosophies*

*Gottlob Frege: Foundations of Arithmetic*

The aim of this book is to introduce and develop an arithmetic analogue of classical differential geometry. In this new geometry the ring of integers plays the role of a ring of functions on an infinite dimensional manifold. The role of coordinate functions on this manifold is played by the prime numbers. The role of partial derivatives of functions with respect to the coordinates is played by the Fermat quotients of integers with respect to the primes. The role of metrics is played by symmetric matrices with integer coefficients. The role of connections (respectively curvature) attached to metrics is played by certain adelic (respectively global) objects attached to the corresponding matrices. One of the main conclusions of the theory is that the spectrum of the integers is " intrinsically curved " ; the study of this curvature is then the main task of the theory. The book follows, and builds upon, a series of recent research papers. A significant part of the material has never been published before.

Making Numbers shares exemplars of good practice drawing on the latest research on using manipulatives to develop understanding of arithmetic. Focusing initially on the teaching of numbers from 1-12, Making Numbers progresses to 200 and beyond, including ideas for teaching partitioning, arrays, and times tables.

L.E.J. Brouwer: Collected Works, Volume 1: Philosophy and Foundations of Mathematics focuses on the principles, operations, and approaches promoted by Brouwer in studying the philosophy and foundations of mathematics. The publication first ponders on the construction of mathematics. Topics include arithmetic of integers, negative numbers, measurable continuum, irrational numbers, Cartesian geometry, similarity group, characterization of the linear system of the Cartesian or Euclidean and hyperbolic space, and non-Archimedean uniform groups on the one-dimensional continuum. The book then examines mathematics and experience and mathematics and logic. Topics include denumerably unfinished sets, continuum problem, logic of relations, consistency proofs for formal systems independent of their interpretation, infinite numbers, and problems of space and time. The text is a valuable reference for students, mathematicians, and researchers interested in the contributions of Brouwer in the studies on the philosophy and foundations of mathematics.

(Longman Library of Primary Sources in Philosophy)

Psychological and Logical Investigations with Supplementary Texts from 1887–1901

Essays on the Foundations of Mathematics and Logic

The Number System

The Basic Laws of Arithmetic

***This 2001 book will appeal to mathematicians and philosophers interested in the foundations of mathematics. The nineteenth century saw a movement to make higher mathematics rigorous. This seemed to be on the brink of success when it was thrown into confusion by the discovery of the class paradoxes. That initiated a period of intense research into the foundations of mathematics, and with it the birth of mathematical logic and a new, sharper debate in the philosophy of mathematics. The Search for Certainty examines this foundational endeavour from the discovery of the paradoxes to the present. Focusing on Russell's logicist programme and Hilbert's finitist programme, Giaquinto investigates how successful they were and how successful they could be. These questions are set in the context of a clear, non-technical exposition and assessment of the most important discoveries in mathematical logic, above all G--ouml--;del's underivability theorems. More than six decades after those discoveries, Giaquinto asks what our present perspective should be on the question of certainty in mathematics. Taking recent developments into account, he gives reasons for a surprisingly positive response.***

***The twentieth century has witnessed an unprecedented 'crisis in the foundations of mathematics', featuring a world-famous paradox (Russell's Paradox), a challenge to 'classical' mathematics from a world-famous mathematician (the 'mathematical intuitionism' of Brouwer), a new foundational school (Hilbert's Formalism), and the profound incompleteness results of Kurt Gödel. In the same period, the cross-fertilization of***

***mathematics and philosophy resulted in a new sort of 'mathematical philosophy', associated most notably (but in different ways) with Bertrand Russell, W. V. Quine, and Gödel himself, and which remains at the focus of Anglo-Saxon philosophical discussion. The present collection brings together in a convenient form the seminal articles in the philosophy of mathematics by these and other major thinkers. It is a substantially revised version of the edition first published in 1964 and includes a revised bibliography. The volume will be welcomed as a major work of reference at this level in the field.***

***Philosophy of Arithmetic***

***Concrete Mathematics: A Foundation for Computer Science***

***Frege***

***The 23rd ICMI Study***

***Gottlob Frege: Basic Laws of Arithmetic***

***From Frege to Gödel***

This volume is a window on a period of rich and illuminating philosophical activity that has been rendered generally inaccessible by the supposed "revolution" attributed to "Analytic Philosophy" so-called. Careful exposition and critique is given to every serious alternative account of number and number relations available at the time.

The Foundations of Arithmetic is undoubtedly the best introduction to Frege's thought; it is here that Frege expounds the central notions of his philosophy, subjecting the views of his predecessors and contemporaries to devastating analysis. The book represents the first philosophically sound discussion of the concept of number in Western civilization. It profoundly influenced developments in the philosophy of mathematics and in general ontology.

Geared toward undergraduate and beginning graduate students, this study explores natural numbers, integers, rational numbers, real numbers, and complex numbers. Numerous exercises and appendixes supplement the text. 1973 edition.

Using Manipulatives to Teach Arithmetic

Philosophy of Mathematics

Essays on Frege's Basic Laws of Arithmetic

Axiomatic Set Theory

Principia Mathematica

Exposition of the System

Gathered together here are the fundamental texts of the great classical period in modern logic. A complete translation of Gottlob Frege's Begriffsschrift--which opened a great epoch in the history of logic by fully presenting propositional calculus and quantification theory--begins the volume, which concludes with papers by Herbrand and by G ö del.

This twenty-third ICMI Study addresses for the first time mathematics teaching and learning in the primary school (and pre-school) setting, while also taking international perspectives, socio-cultural diversity and institutional constraints into account. One of the main challenges of designing the first ICMI primary school study of this kind is the complex nature of mathematics at the early level. Accordingly, a focus area that is central to the discussion was chosen, together with a number of related questions. The broad area of Whole Number Arithmetic (WNA), including operations and relations and arithmetic word problems, forms the core content of all primary mathematics curricula. The study of this core content area is often regarded as foundational for later mathematics learning. However, the principles and main goals of instruction on the foundational concepts and skills in WNA are far from universally agreed upon, and practice varies substantially from country to country. As such, this study presents a meta-level analysis and synthesis of what is currently known about WNA, providing a useful base from which to gauge gaps and shortcomings, as well as an opportunity to learn from the practices of different countries and contexts.

This book explores arithmetic's underlying concepts and their logical development, in addition to a detailed, systematic construction of the number systems of rational, real, and complex numbers. 1956 edition.

The Foundations of Mathematics in the Theory of Sets

A Contribution to the Philosophy of Geometry

A Source Book in Mathematical Logic, 1879-1931

*"There are many textbooks available for a so-called transition course from calculus to abstract mathematics. I have taught this course several times and always find it problematic. The Foundations of Mathematics (Stewart and Tall) is a horse of a different color. The writing is excellent and there is actually some useful mathematics. I definitely like this book."--The Bulletin of Mathematics Books*

*First published in 1982, this reissue contains a critical exposition of the views of Frege, Dedekind and Peano on the foundations of arithmetic. The last quarter of the 19th century witnessed a remarkable growth of interest in the foundations of arithmetic. This work analyses both the reasons for this growth of interest within both mathematics and philosophy and the ways in which this study of the foundations of arithmetic led to new insights in philosophy and striking advances in logic. This historical-critical study provides an excellent introduction to the problems of the philosophy of mathematics - problems which have wide implications for philosophy as a whole. This reissue will appeal to students of both mathematics and philosophy who wish to improve their knowledge of logic.*